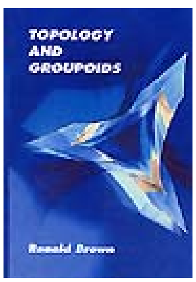
MAA Reviews

Topology and Groupoids

Ronald Brown



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MAA Review

[Reviewed by Michael Berg, on 07/15/2009]

Why groupoids? Well, first one should ask, what *are* groupoids? After all, these things are not found in the standard graduate school curriculum and are not part of most research mathematicians' tool kits. (If Ronald Brown, the author of the book under review, has it right, however, this may very well change in the not too distant future; in fact we're long overdue for it.)

Well, what are they? First of all, they exhibit a fundamentally geometric or perhaps even categorical aspect in that the attendant binary operation (as befits anything "groupie," so to speak) requires for the product AB to defined that the initial point of the arrow representing A coincide with the terminal point of the arrow representing B. Says Brown on p.xx of his book: "This corresponds to the composition of journeys." And he goes on: "Conversely, the analysis of a journey through various places requires precisely this notion of partial composition." And then: "The theory of groupoids has added to group theory a spatial component, coming from the geography of the places we visit in a journey. For this reason, groupoids can model more of the geometry than groups alone."

Surely, this is a marvelous and potentially hugely useful thing, and even the preliminary description given above suggests that groupoids should occur quite naturally in the formulations of certain concepts in, e.g., algebraic geometry, category theory, and —

most emphatically — algebraic topology. Certainly, the image of gluing arrows brings to mind some of Grothendieck's slang concerning composing morphisms, even rising to the level of the "graphical" characterization of the derived category of a given abelian category (see, e.g., Gelfand-Manin, *Homological Algebra*). It is indeed telling, in this context, that Brown closes the aforementioned Preface with the following passage: "I also thank Alexander Grothendieck for an exuberant correspondence in the years 1982–1991: he described this as '*a baton rompu*,' which roughly means 'ranging over this and that,' and indeed it dealt with many matters of mathematics and life."

Furthermore, at the end of the book, in the final chapter entitled "Conclusion," Brown returns to Grothendieck in connection with the pedagogical matter of how to elaborate new concepts in the context of mathematical progress, quoting the latter as referring (*loc. cit.*) to the "difficulty of bringing new concepts out of the dark." Brown then goes on to mention Grothendieck's famous metaphor of the rising sea for a means whereby to solve a mathematical problem: "...not like cracking a nut with a hammer, but more like gradually softening the shell with water till it can be peeled away..." (I believe that in *Récoltes et Sémailles* Grothendieck elaborates the corresponding metaphor of the rising sea eventually to submerge the island representing a given mathematical difficulty, flooding it, as it were, with enough structural machinery to make it appear as a triviality.)

Thus, the style of this book is perhaps somewhat against the grain, in ways Grothendieck would approve of. I think this is fitting in light of the fact that Brown is truly concerned with proselytizing what he believes to be an important new wave in mathematics.

But it turns out that this new wave has been building for quite a while, and it has already begun to sweep a number of scholars along with it in its orbit. Outside of (algebraic) topology, Alain Connes has seen fit to include it in his program of non-commutative geometry (Brown, p. 411), and K. MacKenzie published a book in 2005 titled, *Lie Groupoids and Lie Algebroids in Differential Geometry*, which reminds me that I in fact heard the word "algebroid" for the first time over twenty years ago, in graduate school, spoken by my algebraist office-mate.

This all fits, then, with the fact that groupoids' first ripples go back to (at least?) the 1960's, and this takes us to Ronald Brown's own early work in this area, in the setting of algebraic topology, work which presently blossomed into a ramified scholarship theme stretching over the next four decades. *Topology and Groupoids* is in point of fact the third phase of an ongoing project, in that its first edition was published by McGraw-Hill already in 1968 as *Elements of Modern Topology* and its second edition appeared in 1988 as *Topology: A Geometric Account of General Topology, Homotopy Types, and the Fundamental Groupoid*, published by Ellis Horwood, Ltd. Even as the present third edition is an updated expansion of the second, the former title is particularly descriptive of what Brown has in mind in this project. The focus really falls on the fundamental groupoid "as a necessary and convenient generalization of ... the fundamental group." (p. xii) Additionally, and connected to the preceding, Brown observes that "[t]he exposition given here lends credence to the view that groupoids form a natural context for discussing ... the relation between local and global phenomena." (*loc. cit.*) Indeed, "[i]t thus seems that the notion of groupoid gives a more flexible and powerful approach to the notion of symmetry." In this connection, see also the article by Alan Weinstein (at Berkeley), "Groupoids: Unifying Internal and External Symmetry," arXiv. Math. RT/9602220 (23 January, 1996).

Certainly, therefore, what Brown is doing in *Topology and Groupoids* is at worst worthwhile and more than likely very important . It is appropriate that Brown assumes a particular pedagogical posture in bringing his message out: even as this book is full of material that would be novel to most of us, even topologists, at least as far as methodology is concerned, the intended audience is far broader than specialists and seasoned researchers. Brown seeks to bring this new way of doing things to beginners in the field, too, and this manifestly accounts for the breadth of his coverage of algebraic and geometric topology in the book's 500+ pages.

Specifically, Brown's first five chapters are true topological background material, stretching as they do from the topology of the real line to a discussion of projective ("and other") spaces. It is only in the sixth chapter (p. 201 ff.) that we get to the all-important fundamental groupoid, but thereafter things get off the ground very swiftly: homotopy theory, cofibrations, computing fundamental groupoids (Van Kampen, the Jordan Curve Theorem revisited), covering spaces, orbit spaces, and, indeed, orbit groupoids. A broad palette.

I do believe in the general efficacy of the general categorical approach in mathematics (even if Grothendieck himself was occasionally heard to use the phrase "abstract nonsense"), and I find Brown's philosophy both attractive and convincing. To wit (p. xx): "In mathematics, and in many areas, analogies are not between objects themselves, but between the relations between these objects. We will define many constructions by their relations to all other objects of the same type — this is called a 'universal property' ... All this is the essence of the 'categorical approach,'... a major unifying force in the mathematics of the twentieth century."

Two final observations. The back cover of *Topology and Groupoids* displays a Venn diagram suggesting that, to borrow another word from Grothendieck, the yoga of groupoids should be amenable eventually to include, or engulf, such objects as groups, group actions, bundles of groups (!), and even sets and equivalence relations. This in itself is a very exciting prospect, alone worth the price of admission.

And this brings me to the second point: *Topology and Groupoids* is published by Ronald Brown himself, so buying the book would be a direct contribution to his cause. I think it is an eminently proper cause, entirely worthy of support. The book is well written, indeed it is really a monograph composed by an insider and an expert; it is very serious mathematics presented in a sound pedagogical style: it is a very readable book equipped with fine examples and many exercises; and its impact should be felt beyond the confines of topology, even as topologists should be attracted to this material most strongly.

Topology and Groupoids is an impressive work which should be given a wide circulation.

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