BOOK REVIEW

History of Topology I.M. James (ed), North Holland/Elsevier, Amsterdam, 1999, 1056pp, US$190.50, Dfl375.00, ISBN 0-444-82375-1.

It is not possible in the space of a review to do full justice to this magnificent volume, which combines scholarship, namely the bringing of past trends to current view in the light of later developments, with excellent exposition.

The major emphasis is on what is called in the Preface 'classical topology', rather than on general (point-set) topology, which will be the subject of another volume. Largely this means algebraic topology, including dimension theory, and parts of general topology which have influenced algebraic topology. Some of the articles are written by professional historians of mathematics, and others by historically-minded mathematicians.

It is also valuable as a human story. We have moved away from the Bourbaki idea of the possibility of erecting a final story of the structure of mathematics. The concepts at the root of mathematics are continually evolving and interacting, rather like individuals in an ecosystem. The struggle with the concepts and problems of topology, of 'continuous geometry', is a great story. All from beginners to experts in mathematical research now have an opportunity to pick a way through the origins of the problems which we see as important today. This re-examination of the past, of the roots of the subject, is an essential part of the development of our subject, to see the controversies of the past, to see the questions that were examined, to make sure that the intuitions of the past have been properly expressed in terms of the language of today, to ensure that vital ideas and problems are not lost but are enabled to be tackled again with a new range of tools and concepts.

Also there are many fascinating, even moving, stories on the mathematicians involved. The story of Listing, who originated the term 'Topologie', is one such, and the stories of mathematicians under the Nazis is another, at a more tragic level. Who can failed to be moved by the story of the suicide of Hausdorff and his wife, in 1942, when he was aged 72?

There are forty articles, of which three are by the Editor. The articles cover a very broad range, from personal accounts to detailed histories of particular areas, or fields, such as homological algebra, shape theory, stable homotopy theory and so on. There are also detailed assessments of the contributions of particular mathematicians, and brief assessments of others.

It is interesting to see laid out the slow development of concepts which to us seem so familiar, such as for example those covered in separate chapters on dimension, manifold, homotopy, complex, fibre bundle, triangulation and continuous group. Of course there has to be a chapter on the contribution of Poincaré, and there are chapters also on Listing, Heegard, Brouwer, Dehn, Nielsen, Hopf, Freudenthal, Seifert, as well as summaries on many others.

Even in a work of this size, there have to be omissions. R.H. Fox is well mentioned for his work on knots, but not for his seminal paper on the compact-open topology, a term which itself does not appear in the index. This does seem an omission from the chapter on the interaction of general topology with other areas of mathematics. Fox's paper led to a lot of work on the topology of function spaces, and the widely used notion of a category of spaces 'adequate and convenient for all purposes of topology' [2] is not mentioned.

In the history of fibre spaces, the theorem on the local-to-global property of the CHP (Covering Homotopy Property) played a key rôle, as explained in Zisman's article. However it is not stated that Dyer and Eilenberg felt that Hurewicz' 1955 paper had a gap in a crucial continuity claim. It is interesting that Spanier's book [8] does not prove that a similarly defined function is continuous (in the first edition the function was not even well-defined). This illustrates the difficulty of the area. An elegant version of this theorem, building on Dold's proof, and generalising it to a result on path spaces and 'schedules', is published in [5].

As is explained in several places in this volume, the higher homotopy groups were first defined by Čech, and he submitted a paper on this to the ICM at Zürich in 1932. This paper was withdrawn, since it was proved that these were abelian. This was a disappointment since people were looking for a higher-dimensional version of the fundamental group which bore a relation to homology similar to that of the fundamental group to the first homology group. That is they were seeking non-commutative higher-dimensional structures, in order better to reflect geometric and analytic properties. This disappointment gradually came to seem a quirk of history, and this attitude is reflected in this book. In particular, J.H.C. Whitehead's general programme of algebraic homotopy, of modelling homotopy theory, is seen in this book only as associated with rational homotopy theory.

In fact, Whitehead pursued a non-commutative structure in dimension 2, namely what he called crossed modules. He developed significant work on these, related to work of Eilenberg and Mac Lane on the cohomology of groups in dimension 3. He was also very proud of his description with Mac Lane of homotopy 2-types (then called 3-types) in terms of crossed modules [7].

To give now a personal view, in 1974 Philip Higgins and I found that one could define *homotopy double groupoids*; that these were in a sense equivalent to Whitehead's crossed module of a pair, consisting of the second relative homotopy group, with its boundary and operations of the fundamental group; and that these could be used to prove theorems which led to new calculations. Now one can see, with further work of Loday [6], that there are non-commutative higher homotopy groupoids in all dimensions and this can lead to new calculations in homotopy theory, separate from those referred to in this volume (see for example the references in the web survey article [3]). For example, these methods allow some computations even of homotopy types, and so of actions of the fundamental group on higher homotopy groups - I remember Whitehead saying that this action was one of the fascinations of the early workers in homotopy theory. However, neither of the terms 'groupoid' or 'crossed module' appear in the index, and this in the end, particularly in view of the work of Connes on groupoids [4], and Baues in algebraic homotopy [1], may also come to seem a quirk of history.

It is interesting to recall the atmosphere in the late 1950s in which topology was seen as a central area, a meeting of many fields, in which the student had to master an array of different techniques from Ext and Tor in homological algebra, to free products with amalgamation of groups. The scene has somewhat shifted, and in the reviewer's opinion it is category theory, following on from Eilenberg, Mac Lane, Ehresmann and Grothendieck, which nowadays has a comparable prospect of foundational influence, and applications, in a wide number of areas. Certainly Eilenberg and Mac Lane were proud of the development of Category Theory, and were seen as often as possible at Category Theory meetings. The 1999 Category Theory meeting at Coimbra was very much in honour of Mac Lane, who attended and spoke with vigour.

All this suggests that we cannot be expected to come to a final view, even on the history of topology. Taken on its own terms for the subjects covered, this book is valuable and will repay repeated study for its view of developments by many of those who took part in them, knew those who did, or have deeply studied the history of the subject. It is unfortunate that the price is likely to deter many mathematicians, particularly young ones, from buying it for themselves.

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[1] Baues, H.J., *Algebraic homotopy*, Cambridge University Press, Cambridge, 1988.

[2] Brown, R., Ten topologies for X x Y, *Quart. J. Math.* (2) 14 (1963), 303-319.

[3] http://www.bangor.ac.uk/~mas010/hdaweb2.htm.

[4] Connes, A., *Non commutative geometry*, Academic Press, San Diego, 1994.

[5] Dyer, E. and Eilenberg, S. Globalizing fibrations by schedules, *Fund. Math.* 130 (1988), 125-136.

[6] J.-L. Loday, 'Spaces with finitely many non-trivial homotopy groups', J. Pure Appl. Algebra, 24, 179-202, 1982.

[7] Mac Lane, S. and Whitehead, J.H.C., 'On the 3-type of a complex', *Proc. Nat. Acad. Sci.* (1950) 41-48.

[8] Spanier, E., *Algebraic topology*, McGraw Hill, New York, 1966.

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