Residual flow, bedforms and sediment transport in a tidal channel modelled with variable bed roughness

A.G. Davies *, P.E. Robins

Centre for Applied Marine Sciences, School of Ocean Sciences, Bangor University, Menai Bridge, Anglesey LL59 5AB, UK

Abstract

The frictional influence of the seabed on the tidal flow in shelf seas and estuaries is usually modelled via a prescribed, spatially/temporally invariant drag coefficient. In practice, the seabed exhibits considerable variability, particularly spatially, that should in principle be included in simulations. Local variations in the seabed roughness ($k_s$) alter the flow strength and, hence, local sediment transport rates. The effect of using a spatially/temporally varying $k_s$ is assessed here with reference to a tidal channel (Menai Strait, N. Wales) in which the variability of the bedforms has been monitored using multi-beam surveying. The channel not only exhibits strong tidal flow, but also a residual induced flow that is used here as diagnostic to assess various bed roughness formulations tested in a Telemac model. Tidal simulations have been carried out with both constant and temporally/spatially variable $k_s$, and the predicted residual flow is shown to be sensitive to these representations. For a mean spring-neap (SN) cycle with variable $k_s$, the average residual flow is calculated to be $525 \text{ m}^2 \text{s}^{-1}$, consistent with observations. This residual flow can be recovered using imposed, constant values of $k_s$ in the range $0.15 \text{ m}$ to $0.3 \text{ m}$. The results suggest that the overall, effective roughness of the seabed is less than half of the maximum local roughness due to the dunes in mid-channel, but more than the spatially-averaged $k_s$ value in the channel as a whole by about 50%. Simulations carried out with an $M_2$-alone tide using variable $k_s$ produce a somewhat smaller (by 7%) residual flow of $491 \text{ m}^3 \text{s}^{-1}$. The use of an 'equivalent $M_2$' tide of amplitude enhanced by 7.3% reconciles these estimates. The main contribution to $k_s$ is made by dunes which are modelled using Van Rijn's (2007) formulation subject to an additional 'history effect'. The modelled $k_s$ is found to equal approximately the observed height of the dunes along mid-channel transects rather than half the height as expected. This is attributed to the non-equilibrium nature of the bedforms in the reversing tidal flow, which exhibited shorter wavelength and more symmetrical profiles than dunes in steady flow.

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- Variable seabed roughness
- Sand dune roughness
- Tidal friction
- Induced tidal residual
- Net sediment transport rate
- Menai Strait, N. Wales

1. Introduction

The presence of dunes on a mobile channel bed influences the hydraulic roughness, water levels and flow rates. Most studies of these effects have involved rivers where the slowly varying roughness $k_s$ of the main channel is sometimes used as a calibration parameter to match observed water levels. In contrast, in tidal environments temporal variations in the flow typically occur more rapidly than the associated roughness of the bedforms, requiring 'history' or 'relaxation' methods to define $k_s$. The sophistication with which the seabed can now be mapped on fine spatial scales challenges modellers to improve the representation of bed features in estuarine and coastal area models. The seabed roughness $k_s$ influences the flow strength, turbulent mixing, bed shear stress, sediment flux and, hence, the morphodynamic development of the bed. Since the bedforms contributing to the roughness occur mainly at sub-grid model scales, the parameterization of $k_s$ is a key element in a model’s formulation. Because hitherto the bed roughness in natural settings has often been largely unknown, the common practice has been to assign a constant $k_s$ value over the entire model area. However, the insights provided by acoustic multi-beam (MBES) instrumentation now give modellers a much clearer picture of the true and often highly variable nature of the seabed and set a challenge as to how best to incorporate a more realistic, spatially and temporally varying, description of the roughness in operational models. The fundamental question posed in this paper is whether a spatially/temporally variable $k_s$ formulation provides a realistic description of the bedforms in a channel, while at the same time representing the tidal flow accurately.

The need for better representation of the bed roughness arises both in coastal areas where combined wave and current effects may be present and also in estuaries where salt-masses and biological influences may be significant (Dalrymple et al., 2012). The starting point of the present study, involving a tidal channel with roughness dominated by bedforms, is the empirical predictor of Van Rijn (2007) for non-cohesive sediments. For given local flow conditions this includes roughness contributions arising from sediment grains, small-scale ripples,
mega-ripples and dunes. Van Rijn’s (2007) formulation was tested in a Telemac model of the Dee Estuary, U.K., by Villaret et al. (2011, also 2012, 2013) and its use gave rise to enhanced bed roughness due mainly to dunes predicted in the main estuary channels. Importantly, the inclusion of this spatially/temporally varying ks did not compromise the robustness of the numerical scheme.

The present paper provides a critical assessment of the use of variable ks with reference to a natural tidal channel (Menai Strait, North Wales, U.K.) which exhibits strong reversing flows, a residual flow induced by the tides, a main tidal channel running between sand banks and over scour holes, and variable patterns of seabed roughness including dunes. The present numerical model, which is the most refined one of the Strait to date, has been developed using the Telemac Modelling System (Hervouet, 2007). The tidal flow is driven by free surface oscillations at its open ends that differ in amplitude and phase in a way that induces the pronounced residual flow. This tidal residual is used here as a diagnostic with which to assess the validity of various formulations for bed friction. The sediment transport is predicted for a seabed comprising an evolving mixture of three prescribed sand sizes overlying a layer of immobile material. The predicted ks is fed back into the flow module, giving rise to flow-bed interaction throughout the simulation. The analysis is focused on both idealized spring-neap (SN) tidal cycles and also semi-diurnal (M2) tides alone. In each case the roughness is computed online using Van Rijn’s (2007) formulation for ks, to which is added a new ‘history effect’ governing the temporal development of the dunes. The results are compared with outcomes determined using constant ks values and the validity of the frictional representations is assessed with reference to the peak ebb and flood flow strengths and also to the strength of the residual flow. The sediment transport rates in the channel are also estimated.

The specific questions posed here to assess the applicability of a spatially/temporally variable ks include: What values of constant ks give rise to the same bulk flows in the channel as predicted using variable ks, and can the full SN-cycle be represented for simplicity by an ‘equivalent’ M2 tide to reduce computational time? Does the mid-channel, maximum, roughness serve as a suitable calibration parameter to represent the channel as a whole or is a cross-channel average value more appropriate (Paarlberg et al., 2006, 2010)? Does the observed dune roughness in the channel agree with Van Rijn’s (2007) predictor or does the fact that this predictor was determined for unidirectional river flows cause it to over-predict ks for reversing tidal flows?

The organization of the paper is listed here. In Section 2 existing knowledge of the seabed roughness is assessed with emphasis on tidal environments. In Section 3 the study site (Menai Strait, N. Wales) is discussed highlighting the residual flow and earlier estimates of the seabed roughness. The Telemac model is described in Section 4 and here the procedures used to model the temporally/spatially variable ks are explained. In Section 5 model outcomes are presented based on both constant and also temporally/spatially variable ks for i) idealized spring-neap (SN) cycles and ii) semi-diurnal (M2) tides alone. The sensitivity of the residual flow to these representations of ks is assessed and an M2 tide ‘equivalent’ to the full SN-cycle is identified. In Section 6 the predicted roughness ks is compared with intra-tidal observations of the dunes, allowing an assessment of the roughness prediction scheme. The conclusions are presented in Section 7.

2. Seabed roughness

The presence of dunes on a mobile channel bed influences the hydraulic roughness, water levels and flow rates. The nature, mechanisms of formation and classification of dunes have been addressed by, e.g., Van Rijn (1993) and Fredsøe and Deigaard (1992). In steady flow in the absence of waves the profiles of dunes are roughly triangular, with a downstream leeside slope equal to the angle of repose above which a separation ‘roller’ occurs. The length of dunes in steady flow is strongly related to the water depth h, typically with \( \lambda_d = 7.3h \), though with observed values of \( \lambda_d \) in the range 3h to 15h. Where bed-load dominates, the height \( \eta_d \) and length \( \lambda_d \) of dunes are related by \( \lambda_d \approx 16\eta_d \). Due to the small bed slope of the upstream face, Fredsøe (1979) argued that formulae derived for sediment transport on a plane bed can still be used since the effect of gravity is small. Dune height in relation to water depth depends upon the transport stage and the median grain size \( d_{50} \) through \( d_{50}/h \) with dune form being complicated by three-dimensional effects in natural channels (Parsons et al., 2005). Where the suspended load becomes significant, \( \lambda_d \) increases in relation to \( \eta_d \) and bedform fields become less uniform (Venditti et al., 2016). Maximum values of \( \eta_d/h \) are about 0.2 in shallow flow of depth 1 m and about 0.1 in deeper flow of 10 m. The morphodynamic evolution of dunes in steady flow has been modelled recently using a 2-D approach by Doré et al. (2016).

Many studies of dunes have been carried out in rivers (Best, 2005) where the roughness of the main channel is sometimes used as a calibration parameter to match observed water levels. Several researchers have proposed methods to forecast river dune dimensions (see Paarlberg et al., 2006) with some attempts made to include the observed time-lag between dune dimensions and flow intensity (e.g. Allen, 1976; Fredsøe, 1979). However, Paarlberg et al. (2006) noted the inability of earlier models to predict the dune dimensions observed during river flood events. Such models have the further drawback of determining the friction factor via the dune dimensions, rather than involving a direct formulation for the dune roughness \( k_d \).

In tidal flow in estuaries Dalrymple et al. (2012) suggested that dunes are somewhat smaller in scale than in steady flow, having \( \lambda_d \approx 5h \) (where \( h \) is the depth corresponding to peak tidal flow) and with \( \eta_d/h \approx 0.2 \) as in steady flow. Van Rijn (1993) noted that the bedforms found most frequently in tidal currents and estuaries are mega-ripples and asymmetrical dunes, with \( \eta_d/h \) in the range 0.1 to 0.15, with smaller bedforms of height \( \eta_d < 1 \) m exhibiting asymmetry changes with the turn of the tidal currents. Zornsd et al. (2011) observed such dune heights and asymmetry effects in the tidal river Elbe. Dune lengths are smaller than those in steady river flow because the tidal period is not long enough for the generation of equilibrium values. Dunes generally occur in deeper wider tidal channels with bed material sizes larger than about 0.3 mm. Van Rijn (1993) stated that tidal bedforms include:

i) mega-ripples in shallow channels with \( d_{50} < 0.3 \) mm; \( \eta_d = (0.05–0.1)h; \lambda_d = (0.5–2)h \)

ii) sand dunes in deeper channels with \( d_{50} = 0.3–0.6 \) mm; \( \eta_d = (0.1–0.2)h; \lambda_d = (3–6)h \)

However, the formulation of Van Rijn (2007) used later also includes dunes in finer sediment (see Fig. A1).

Flow resistance due to dunes arises from the combined effect of skin friction and form drag due to the expansion loss after each crest. This loss was estimated using Carnot’s formula by Engelund (1978) and Fredsøe and Deigaard (1992), and the latter authors presented a flow resistance curve for dunes, based on similarity principles, that indicates the predominance of form drag at intermediate flow stages (see also Niemann et al., 2011).

For dunes defined as asymmetrical bedforms with length of about 7 times the water depth, Van Rijn (1993) proposed the following rule for the dune roughness:

\[
k_{sd} = 1.1 \gamma_d \eta_d \left(1 - e^{-25\lambda_d/\lambda_d}ight)
\]

where \( \gamma_d \) is a form factor equal to 1 for angle-of-repose lee slopes and 0.7 for less steep slopes in field conditions. However, Van Rijn (1993) noted that a considerable reduction in the form roughness arises for dunes having mild leeside slopes above which separation is less important. This has been highlighted recently by a series of laboratory experiments carried out by Kowell et al. (2016). With the horizontal
length of the dune lee slope taken as $\lambda_d$ Van Rijn (1993, Fig. 6.2.8) indicated that, for values of $\alpha_d/\eta_d$ greater than about 10 (i.e. gentle lee slopes), $\gamma_d$ takes values smaller than 0.2. Li et al. (2006) suggested an improvement on Van Rijn's form factor based on FLUENT simulations which suggested an even more pronounced reduction in $\gamma_d$ as $\alpha_d/\eta_d$ increases. Lefebvre and Winter (2016) noted that angle-of-repose bedforms are rarely found in the field and highlighted the effect on $k_{sd}$ of lee slope angle based on simulations made using Delft3D. However, the Reynolds-averaged model that they used did not resolve turbulence and intermittent flow structures, only permanent flow separation, resulting in a drastic reduction in $\gamma_d$ between slope angles of 30° and 20°, with $\gamma_d$ becoming $<0.01$ at angles of $<15°$. The severity of this reduction in $\gamma_d$ is not easily reconcilable with the separation zones observed by Lefebvre et al. (2013) over lee side slopes of (10–20°). However, the detail of the bedform shape adopted has been shown recently to have influenced the model outcomes (Lefebvre et al., 2016). Observations of the velocity and turbulence fields were made above a ’sandwave’ in a tidal estuary by Atkins et al. (1989). The field site was uncovered at low tide and then experienced flood-dominated flow with the depth reaching 4–5 m at high tide. The resulting bedforms in fine sand ($d_{50} = 0.2 \text{ mm}$) had wavelengths of 15–20 m and heights of 0.8 m. In terms of Van Rijn’s form factor the ’standard sandwave’ had $\lambda_d/\eta_d$ in the range 3.3–3.8 indicating a value of $\gamma_d > 0.9$ and, hence, large roughness $k_{sd}$. The separating flow over these dunes was simulated by Johns et al. (1993), and a comparison with these data was carried out also by Li et al. (2006).

Paarlberg et al. (2006, 2009) suggested a morphodynamic modelling approach for the prediction of dune dimensions involving cutting off the separation zone above the dune lee slope and computing only the steady flow above this (see also Van Duin et al., 2017). The shape of the zone was related to the dune dimensions, with flow separation occurring for leeside slopes exceeding 10° and with the flow then reattaching downstream at a typical distance of 5 to 6 times the dune height (cf. Fredsøe, 1982). The model was used to predict dune evolution, including asymmetry, and Paarlberg et al. (2010) later translated the estimated dune dimensions into roughness coefficients using Eq. (1). The changing dune dimensions gave rise to hysteresis in $k_{sd}$ as observed during river flood events in which maximum dune heights occur a few days after peak discharge (Paarlberg et al., 2010). During such events the primary dunes cannot adapt instantaneously to the changing flow conditions. This can lead to a reduction in $\lambda_d$ via the formation of secondary dunes, and also shorter and lower dunes (Wilbers and Ten Brinke, 2003), the composite effect of which can be included in the roughness formulation (Julien et al., 2002). Paarlberg et al. (2010) also noted the pronounced cross-channel changes in roughness caused by variations in grain size and current speed.

In reversing tidal flows these considerations remain, with the flow also being modulated through the SN-cycle. Bartholomá et al. (2004) made field observations of bedform behavior within individual ebb-flood tidal periods in the Grådyb channel, Denmark. While conditions at their study site appear somewhat similar to the channel considered in this paper, they reported that the bed was covered with very large compound dunes (length about 100 m, height 1.5–3 m) with superimposed medium-sized simple dunes. Only the latter features are present in the Menai Strait, apart from at its southwest end where some larger features occur. Bartholomá et al. (2004) noted that dune heights showed considerable variation over the tidal period with the smaller superimposed dunes controlling the transport processes.

Paarlberg et al. (2010) noted that tidal dunes are more symmetrical than those in rivers, with the roughness description requiring adaptation on this account. A similar conclusion was reached by Lefebvre et al. (2011) who measured large compound bedforms through a tidal cycle in the Jade tidal channel, Germany. They found that published roughness predictors were unable to estimate not only the variations of the primary dune roughness but also tidally-induced changes in roughness for smaller secondary dunes. Lefebvre et al. (2013) made further roughness estimates from data obtained in the Knudedyb tidal inlet, Denmark. Here during the ebb tide they found that a permanent separation zone was established over the steep (10–20°) lee sides of ebb-oriented primary bedforms, whereas during the flood no separation was induced over the gentle (2°) lee slopes. In a modelling exercise using Delft3D, Lefebvre et al. (2014) investigated the presence and length of the flow separation zone in comparison with a predictive method (Paarlberg et al., 2009), with at least qualitative agreement achieved.

Allen (1976), among others, noted that dunes in river-dominated settings ‘lagged behind the flow’ and had a ’relaxation time’ in excess of the time scale of change of the flow. This relaxation time, which can vary from hours to several days depending upon the scale of the system, is the time needed for the dunes to equilibrate with a changed flow. Van Rijn (1993) suggested adjustment rules for the evolution of $\gamma_d$ and $\lambda_d$ involving a ’transition period’ and Fredsøe (1979) explained the relaxation/transition time lag in terms of the expansion loss behind dune crests. Dairymple et al. (2012) suggested further that profile asymmetry is strongly dependent on the size of a dune since the lag time increases approximately as the square of the dune size. They argued that in estuaries, while small simple dunes can reverse partially or completely during each half tidal cycle, large to very large compound dunes may require months to do so and, thus, can provide an indication of the residual transport direction over such periods. Kostaschuk and Best (2005) noted that, in addition to the morphology of dunes in estuaries often lagging behind changes in SN-tides, changes in dune morphology also occur during the semi-diurnal tidal cycle. This effect was monitored in the Main Channel of the Fraser River estuary in water of depth 12–15 m with mean velocities of up to 2 m s$^{-1}$. Dune length did not change significantly during the 13-hour survey period, but changes did occur in dune height, steepness and leeside slope angle. Hysteresis loops indicated that $\gamma_d$ was in equilibrium with strong flows during the tidal cycle, but that $\lambda_d$ was not, requiring longer to respond to the changing flow conditions.

The present study adopts Van Rijn’s (2007) procedure to calculate $k_s$ directly from the predicted sediment grain size, flow strength and water depth. This procedure, derived for steady flow, is accompanied by a new simple relaxation time, or ‘history effect’, designed to prevent unduly rapid variations in $k_s$ during tidal cycles, but still allowing a slow modulation in $k_s$, during the SN-cycle. Coleman et al. (2005) proposed a similar power law ’history effect’ relating bedform wavelength and height to time with all parameters normalized by equilibrium values, starting from either plane bed or an initial bedform state. The roughness predicted in Section 5 is dominated by the dune component $k_{sd}$ which is critically assessed in Section 6 through observations made in the Menai Strait of dune dimensions, asymmetry and migration. This comparison highlights some of the differences noted above between dune occurrence in reversing tidal flows and more slowly varying river flows.

3. Study site: Menai Strait, N. Wales

The Menai Strait, which separates the island of Anglesey from the mainland of Wales, has the geological and sedimentological setting discussed by Roberts et al. (2011). The Strait is approximately 20 km long with mean width of about 800 m between locations A Fort Belan and C Bangor Pier (Fig. 1). The volume of water within this length of the Strait is about 80 million m$^3$ at mean tidal level. Tidal ranges in the Strait are large, being of similar magnitude to the mean depth at some locations; the $(M_s, S_2)$ tidal constituent amplitudes are, respectively, (1.6 m, 0.5 m) and (2.5 m, 0.75 m) at marked locations A and D in Fig. 1a at the open sea ends of the Strait. Tidal elevations at these locations are shown in Fig. 2a. The instantaneous flow is driven by the larger tidal range at the northeast end of the Strait than at its southwest end, with a phase lag in elevation of about 40 min at the
northeast end due to the time of travel of the tidal wave around Anglesey and into Liverpool Bay.

The tidal dynamics in the Strait are controlled by the shallow, central area of the Swellies (Fig. 1c), where the minimum flow cross-sectional area is located. Here, in contrast to the rest of the Strait, relatively large along-channel variations in tidal range and phase occur over a short distance. When high tide occurs in the Swellies, the flow towards the southwest is at its strongest, augmented by the reduced frictional resistance occurring for the deeper flow; in contrast at low tide, the frictional resistance is enhanced giving rise to a substantially weaker, shallow flow towards the northeast. The net effect is a residual flow towards the southwest with magnitude modulated by the stage of the SN-cycle. Early observations of the flow showed maximum instantaneous velocities of about 0.8 m s⁻¹ towards the northeast and 1.5 m s⁻¹ towards the southwest (Harvey, 1968), with a mean velocity of about 0.11 m s⁻¹ occurring towards the southwest in conditions involving light winds.

Harvey (1968) took account of intra-tidal elevations in estimating water fluxes at four sections across the Strait and determined a southwesterly residual flow of magnitude up to 38 million m³ in a
12½ hour period. Although Harvey (1968) did not fully identify the SN-tidal variation, his data obtained for the highest and lowest tidal ranges suggested a variation in the residual flow rates over a 12½ hour period from about 29 to 38 million m³, corresponding to about 644 to 844 m³ s⁻¹. Since Harvey (1968) took no account of cross-channel variation in velocity, his residual flow calculations based on mid-channel velocity observations were probably overestimated (see Section 5.2). Nevertheless his values corresponded to a significant proportion (~1/3) of the total volume of water within the Strait and suggested that the system has a flushing time of 2 to 3 days. Fresh water inflow into the Strait has been estimated as <0.5 million m³ during a semi-diurnal cycle and is therefore unimportant. Harvey (1968) noted, however, that variations in the southerly wind component were likely to bring about variations in the residual flow.

Simpson et al. (1971) investigated the effects of wind stress and tidal range on the residual flow using electromagnetic measurements made at a central transect. Their estimate of the net water transport over a spring tidal cycle was 48 × 10⁶ m³ in a 12½ hour period, which is somewhat larger than Harvey’s (1968) value. The component of wind stress acting along the Strait was shown to augment the residual flow when from the northwest and to reduce, or even reverse it, when from the southwest. Regression analysis over a 4-month period suggested, for conditions of negligible wind stress, that the residual transport was in satisfactory agreement with Harvey’s (1968) estimate. In the absence of wind, Simpson et al. (1971) estimated the residual flow to be about 800 m³ s⁻¹ at mean springs and 330 m³ s⁻¹ at neaps.

Observations of the along-channel water surface slope were made in a central part of the Strait by Campbell et al. (1998) to explore the dynamical balance and deduce the seabed drag coefficient CD (bed shear stress τb = ρCDU², ρ = water density, U = depth-mean velocity). The water column was shown to be vertically homogeneous, with horizontal density gradients too weak to affect the dynamics. For two periods of observation, the total transport of water, calculated from the cross-sectional area and the sectionally-averaged velocity, was respectively 26 and 30 million m³ every 12.42 h (or 578 and 667 m³ s⁻¹), the latter value being for a period with stronger currents. The corresponding values of CD determined along the centre-line of the Strait were 0.0088 and 0.0084, both values much larger than normally used (~0.0025) in shelf-sea applications. Assuming a log-velocity profile over the full water column and with the observed depths in mid-channel in the range 13 m to 17 m, the roughness ks implied by Campbell et al. (1998) varies from 1.8 to 2.6 m, which is unexpectedly large for flow in a sandy channel.

Further measurements of velocity and turbulence were made by Rippeth et al. (2002) over spring and neap tidal cycles at a closely adjacent site to that of Campbell et al. (1998) again in water of mean depth 15 m. The ‘variance method’ was used to extract the bed shear stress from ADCP turbulence data yielding a much lower value of CD = 2.6 ± 0.2 × 10⁻³. Due to uncertainties in their stress estimates, the authors suggested that these values could be increased by ~11% making CD = 0.0029, corresponding to ks = 0.10 m based on an assumed logarithmic profile over the full depth. In contrast, estimates of the friction velocity u* (τb = ρu*u*) from the log-profile method produced larger values of the stress and, by implication, larger values of CD. These CD values were smaller on ebb (0.0032) than flood (0.0042), and smaller on neaps (0.0034) than springs (0.0040) (corresponding to ks values of 0.14 m, 0.35 m, 0.17 m and 0.30 m, respectively). Rippeth et al. (2002) suggested that the difference between the estimates from the variance and log-profile methods might arise from the variance method measuring only the skin frictional drag while the log-profile also incorporates contributions due to larger scale form drag. However this explanation would only have force if the bed features were of relatively short wavelength in relation to the height range over which the log-fitting was carried out and the authors did not comment on the bed type and size on site. The smaller CD values on the ebb than the flood could have been caused by the stronger ebb (southwesterly) flow generating dunes with asymmetrical profiles (steep face downstream). The weaker flooding (northeasterly) flow might then have experienced these features as an enhanced roughness (larger CD). The observed SN-variation might indicate greater dune size on springs. All of these conjectures are assessed later.

Further consideration was given to the drag coefficient CD through a ‘tide-watch’ (George, 2006). This was carried out over a semi-diurnal, mean spring tide at nine locations, and the outcomes were used to evaluate CD for a series of reaches along the Strait. For the reach considered by Campbell et al. (1998) and Rippeth et al. (2002), George (2006) obtained CD = 0.0075. However, in a subsequent modelling study, George (2007) used the tide-watch data to suggest that a spatially varying CD gave a reasonable fit between the modelled and measured data.
with: \( C_0 = 0.0035 \) southwest of the Swellies; 0.0050 in the Swellies; and 0.0020 northeast of the Swellies.

George (2006) noted the distorted nature of tidal curves southwest of the Swellies, where the tide falls approximately linearly for the first 2½ hours after high water and then rather faster. This behavior was attributed by Bowers et al. (2013) to a seiche that can produce a point of inflection in the tidal curve or a double high water.

### 4. Model implementation with bed roughness feedback

#### 4.1. Telemac model – implementation and validation

The simulations were carried out with the Telemac system (mainly v6p3, but also v7p1), based on finite-element algorithms, which allows complex coastal areas to be represented on an unstructured numerical grid (Hervouet, 2007; Villaret et al., 2013). The depth-averaged hydrodynamic module Telemac2D, which simulates the tidal flow in the Strait, was coupled with the morphodynamic module Sisyphe, which calculates sediment transport rates decomposed into bed load and suspended load. A key feature of the present implementation was the prediction by Sisyphe of the changing roughness \( k_s \) due to the combined effects of sand grains, small-scale ripples, mega-ripples and dunes. The details of the bed roughness scheme, which were outlined by Villaret et al. (2011), are summarised in Section 4.2 and Appendix A, together with the new ‘history effect’ applied to the dune roughness component \( k_d \).

The model domain includes the Menai Strait and coastal areas at both its northeast and southwest ends (Fig. 1). The unstructured grid comprising 131,800 computational nodes was created using the BlueKenue™ grid generation software (Canadian Hydraulics Centre, 2011). The grid has variable resolution, from fine (\( -20 \) m) in the central channel of the Strait (Fig. 1b) to coarse (up to 500 m) elsewhere. This grid was mapped onto an assemblage of bathymetric data comprising: (1) multi-beam (MBES) data of the channel below mean low water springs, at high horizontal resolution (\( 5 \) m), collected during 2012; (2) LiDAR data of terrestrial and intertidal areas above the level of low water springs, at high horizontal resolution (\( 2 \) m), collected during 2013; and (3) Admiralty bathymetric data of the offshore regions at either end of the Strait (interpolated onto a 200 m horizontal resolution grid). The amalgamated bathymetric mesh (Fig. 1) represents a significant improvement on grids used in previous studies. The model time-step was 2.5 s, and the model output was stored every 10 min (600 s).

A baseline validation was performed using Telemac2D alone to check that the model could reproduce the tidal dynamics. The model was forced at each open boundary (i.e. at the ends of the Strait) by the five principal tidal constituents: semi-diurnal lunar (\( \text{M}_2 \)) and solar (\( \text{S}_2 \)), together with \( \text{N}_2, \text{K}_1, \) and \( \text{O}_1 \), all obtained from a regional-scale model of the Irish Sea (Lewis et al., 2015). The seabed friction was represented by a constant roughness \( k_s = 0.32 \) m, a value determined by Bowers et al. (2013) to a seiche that can produce a point of inflection in the tidal curve or a double high water.

#### 4.2. Sediment transport and bed roughness prediction

The sediment transport rate was calculated using the formulation of Bijker (1992), with seabed evolution then calculated by solving the Exner equation, including sloping bed effects (see Davies and Thorne, 2008; also Mendoza et al., 2017). For simplicity the suspension procedure available in Sisyphe was not utilized, i.e. advection-diffusion was not used for the suspended sediment. The transport procedures were implemented in the coupled model assuming a seabed composed (initially) of a representative upper layer of three sediment sizes (0.125, 0.25 & 0.5 mm) in the volumetric proportions 30%, 50% and 20%, respectively, overlying a layer of non-erodible cobbles of size 10 mm. The artificial of a coarse underlying layer is particularly important in the shallow region of the Swellies where strong currents would otherwise give rise to unrealistic bed erosion. The thickness of the initial, erodible, upper layer was chosen as 30% of the local water depth, subject to minimum and maximum assumed values of 0.5 m and 5 m, respectively, and with an initial erodible layer thickness of 1 m imposed in the Swellies. As a typical simulation proceeds, the sediment mixture in the upper layer is redistributed due to differential transport of the component grain sizes and the cobbles can become exposed in places. The evolving sediment mix in the upper layer determines the mean grain diameter used in the roughness prediction scheme which, in turn, influences the flow strength. The interactive formulation is therefore quite complex and the simulation time correspondingly lengthy.

A key part of the present coupled simulation involves the seabed roughness, which was predicted by the model, rather than being specified as a global value for the entire domain (cf. the validation in Section 4.1). In nature, the dimensions of bedforms vary spatially and with time, as a function of the fluid field and sediment characteristics (Section 2). The time-varying total roughness \( k_s \) was calculated here at each grid node by Sisyphe and then used by Telemac2D. The total bed shear stress was partitioned in Sisyphe such that only the small-scale ripple roughness was used for local sand transport predictions. The feasibility of this method of feedback between the hydrodynamic and morphodynamic models was demonstrated previously in estuarine applications (e.g., Villaret et al., 2011), with the methodology appearing robust and without unstable feedbacks occurring between the mean flow and the bedforms.

The method of bed roughness prediction and the modelling methodology are presented in Appendix A. Van Rijn’s (2007) empirically-based

### Table 1

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<tr>
<th>Tidal Gauge Station</th>
<th>( M_2 )</th>
<th>( S_2 )</th>
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\( H \) = tidal amplitude (m); \( g \) = phase (degrees).
formulation for \( k_s \) is simply applied and offers a physically consistent approach to represent the sub-grid-scale processes; typical dunes having wavelength \( O \) (10 m) are here represented as ‘roughness’, rather than ‘topography’, the minimum local grid size being \( \sim 20 \) m. The formulation predicts the local bed roughness only and does not permit any lateral spreading of dunes along their crest lines, as tends to occur on site towards the shallower margins of the Menai Strait, i.e. no diffraction-like effect is modelled here.

The predicted bed roughness depends upon a single parameter, namely the mobility number, and includes contributions due to small-scale ripples \( (k_{sr}) \), mega-ripples \( (k_{smr}) \) and dunes \( (k_{sd}) \). The formulation does not involve the prediction of the dimensions (height and wavelength) of the respective bedforms since Van Rijn argues that this is an unnecessary complication. Instead, direct estimates are made of the roughness components that are then used to calculate both the bed friction encountered locally by the mean flow and also the local sediment transport rate. Initially, \( k_{sr} \), \( k_{sm} \) and \( k_{sd} \) are computed quasi-steadily, and are then combined into a total bed roughness \( k_{s,TOT} \), taking no account of the ‘history effects’ that occur in reality in a tidal flow. The limitations of a simple quasi-steady approach were illustrated by McCann et al. (2011) who compared model results for \( k_s \) with observations of bedforms made in a shallow estuarine flow. The model results for \( k_s \) fluctuated strongly on a short tidal time scale, while the observations only exhibited smaller, damped, temporal variations. This suggests that, while a quasi-steady approach can illustrate the effects of spatial variation in \( k_s \), it can misrepresent some potentially important aspects of roughness ‘feedback’ into the dynamics of the flow.

The aim of the new ‘history effect’ included to address these shortcomings is to inhibit the rate of bedform growth/destruction in a realistic manner. Only the dominant dune roughness component is adjusted here, though the same principle and methodology may be applicable also to mega-ripples. A time scale \( T_D \) for the growth/destruction of a simple dune (Section 2) is suggested as: \( T_D = U/d \) where \( d \) (\( \equiv d_{s0} \)) is the sediment grain size and \( U \) the strength of the flow. So the larger are the grains the longer is \( T_D \), and the larger is the velocity the shorter is \( T_D \). The time of formation/destruction of fully-developed dunes can then be expressed via a calibration constant \( \gamma_s \), as follows: \( T_D = \gamma_s \times T_D \) where \( \gamma_s \) is \( O(10^3) \). This time scale \( T_D \) is assumed here to provide the simplest basis upon which to define a lag effect, or ‘history effect’, for dune growth or erosion. If at a particular grid location and model time step the quasi-steady approach to the calculation of \( k_{s,TOT} \) implies a dune growth/destruction rate that is faster than that which can possibly be attained according to the time scale \( T_D \), then the growth/destruction rate of the dunes is inhibited. The procedure is explained in Appendix A.

5. Model results

5.1. Hydrodynamic and sediment transport simulations

A central aim was to use the validated model to simulate the tidal flow across the Flux transect (see Fig. 1b), and hence to calculate the net water flow through the channel at this location. To this end, 20-day simulations were carried out to include a mean SN-tidal cycle (i.e., 14.75 days based only on the M2 and S2 tidal constituents). No wind or wave effects were included and \( k_s \) was predicted by the model. The tidal elevations at the ends of the Strait are shown in Fig. 2a, together with the resulting elevation at the middle of the Flux transect. This is somewhat larger in amplitude and almost in phase with the elevation at the southwest end of the channel (location A). Fig. 1b shows the numerical grid in the vicinity of the Flux transect; here the grid resolution of nominally \( \sim 20 \) m can be seen in relation to the transect, together with the mid-channel point (marker). The tidal range at this point is \( \approx 4.7 \) m on springs and \( \approx 2.5 \) m on neaps; the tidal range is significantly smaller on the southwest side of the Swellies than on the northeast side. A time series of depth-averaged velocity at the mid-channel point is shown in Fig. 2b. The peak spring (negative) ebb depth-averaged tidal velocities directed southwards are stronger (up to \( 1.35 \) m s\(^{-1} \)) than the (positive) flood velocities (up to \( 0.8 \) m s\(^{-1} \)) directed northwards. The peak neap currents are weaker, reaching \( 0.8 \) m s\(^{-1} \) during the ebb and \( 0.5 \) m s\(^{-1} \) during the flood. Strongest flows occur in the southerly direction because, at this stage of the tide, water depths are greatest due to the tidal forcing at the northeast open-sea boundary. Because of the consequent flow speeds at high and low water, there is a significant net flow through the channel from its northeast to its southwest end, i.e., southward across the Flux transect.

Based on the defined mixture of bed sediment sizes (Section 4.2), the predicted roughness \( k_s \) during spring tides (Fig. 3a) suggests the presence of dunes in mid-channel, with rather featureless beds towards the shorelines, as observed. At the location of the Flux transect, the dunes achieve heights (assumed here to equal \( k_s \)) of about \( 0.6 \) m in the main channel, while towards the shores the roughness corresponds to small-scale sand ripples or featureless rough beds. These latter areas occur particularly in the shallower waters towards the eastward end of the transect. The roughness \( k_s \) shows relatively little SN-variation once the dunes have become established; after ‘growing’ during the first couple of days, they are predicted to persist with only small modifications thereafter due to the imposed ‘history effect’ (Appendix A). The time series (Fig. 3b) of \( k_s \) at the mid-channel point shows both intra-tidal temporal changes and also a systematic, albeit small, SN-variation.

The mean sediment grain size varies in the surface (upper) bed layer during a simulation. After 20 days, coarser sediment (exposed cobbles from the non-erodible lower layer) is predicted to occur (Fig. 4) in the shallow water at the edges of the Strait with finer sediment in the main channel, as observed. This redistribution is probably enhanced by any localised wave effects at the margins of the channel. The grain-size texture predicted for the surface layer is shown in more detail in the inset. Again, the finest sediment occurs in the main channel with a coarsening tendency on either side.

Based on the 3-sediment mixture, Fig. 5a shows the predicted magnitude of the sediment transport rate (in \( m^2 \) s\(^{-1} \)) at maximum spring tide with the flow towards the south giving rise to relatively large transport rates in the centre of the channel. Towards the shorelines, little sediment movement is predicted. In contrast, the transport to the north on spring tides is very small. At the mid-channel point, the sediment transport (plotted in Fig. 5b as a scalar) is predominantly southwards (large peaks) with only small intervening northerly transport contributions (small peaks). The resulting net sediment flux is southwards, and so its direction is the same as that of the residual flow discussed in Section 5.2.

All of the results above are for a mean SN-cycle (\( M_2 \) and \( S_2 \) only). Variations around this typical mean pattern will occur in practice due to the presence of smaller tidal harmonics not included in the simulations.

5.2. Water flux results

The residual flow across the Flux transect is used here as a diagnostic to assess various bed roughness descriptions implemented in Telemac2D via Sisyphé. Firstly, an idealized SN-tidal cycle was simulated with i) predicted spatially/temporally varying \( k_s \) and ii) several fixed values of \( k_d \) (0.05, 0.15, 0.30 and 0.45 m) throughout the domain. Each run was carried out for a duration of 20 days allowing a spin-up period of about 2 semi-diurnal cycles for the flow and, where needed, about 3 cycles for the variable \( k_s \), before commencing the averaging process to extract the residual flow. Secondly, an \( M_2 \)-alone cycle was simulated to assess whether a simple ‘equivalent \( M_2 \) tide’ can be defined to represent aspects of the full SN-cycle in order to reduce computation time. To this end a series of \( M_2 \)-alone runs was performed, again with i) varying \( k_s \) and ii) fixed values of \( k_d \) (0.05, 0.10, 0.20, 0.30, 0.40 and 0.50 m). The \( M_2 \)-alone simulations were of duration 50 h and were run for convenience with a tidal period of 12.5 h. In these runs the flow
and bedforms became established on the same time scales as for the SN-cycle allowing flux averages to be obtained during the final 12.5 h.

The instantaneous, north-south, \( v \)-component of velocity was calculated on the grid shown in Fig. 1b and then interpolated in BlueKenue onto the west-east transect of length 365 m using a regular interval of 5 m. The predicted variable \( k_s \) across the transect at the end of the 50-hour \( M_2 \)-alone simulation is shown in Fig. 6. Peak values of \( k_s \approx 0.6 \) m occur in the main channel where dunes are predicted (cf. Fig. 3) with smaller values of \( k_s \) towards both shores. The effect of variable versus fixed \( k_s \) on the \( v \)-velocities across the transect is illustrated in Fig. 7a. Fixed \( k_s \) gives rise to smaller flow speeds near both shores where the variable \( k_s \) tends to be smaller. In contrast in the main channel the dune roughness gives rise to a flow speed that falls between the values obtained with fixed values of \( k_s = 0.05 \) m and 0.5 m. With low constant \( k_s \) (light dashed line in Fig. 7a for which the value imposed is equal to about the minimum value predicted along the transect), velocities are greater in mid-channel than for variable \( k_s \). With high constant \( k_s \) (dark dashed line in Fig. 7a) the water flux is reduced as expected, with lower flow speeds found in mid-channel than for variable \( k_s \), even though \( k_s = 0.5 \) m is lower than the predicted \( k_s \) value in mid-channel. Close to the shore, flow speed variations arise from overall water flux constraints, with even a flow reversal predicted on the eastern shore for small constant \( k_s \) values, which is not due to wetting and drying effects according to Fig. 7b. The use of a fixed \( k_s \) necessarily provides overall qualitative agreement with the variable \( k_s \) solution. However, the differences in the predicted velocities are actually quite large, and therefore capable of triggering potentially different morphodynamic outcomes. It should be noted that during the \( M_2 \)-alone simulations the bed evolution predicted by Sisyphe was fed back into Telemac2D. However, the water depth changes on this account were too small to have affected the results shown in Fig. 7a significantly.

Simpson et al. (1971) estimated that the mean (cross-sectionally averaged) velocity in spring and neap conditions at a neighbouring transect across the Strait was approximately 0.87 times the maximum, mid-stream, depth-averaged current velocity. This calculation was repeated here using the model results for variable \( k_s \) obtained at the Flux transect. For tidal phases corresponding to peak velocity northward and southward the quotients relating the mean and peak depth-averaged velocities were 0.86 and 0.81 respectively, i.e. somewhat smaller than Simpson et al.’s (1971) estimate. The magnitude of the quotient varies in time around these representative values which were obtained during rising and falling tides respectively.

The water fluxes were determined in BlueKenue by forming the product of the \( v \)-component of velocity and the instantaneous local water depth \( h \) at each point across the Flux transect. These 74 values of \( h \cdot v \) at 5 m spacing were integrated to give the instantaneous water flux every
10 min. The results for the SN-cycle with variable \( k_s \) are shown in Fig. 8a. Following an initial model spin-up period, the water flux shows a predominant M2 oscillation modulated by S2 with larger peak fluxes southward and smaller peak fluxes northward. The southerly (negative) residual was extracted through the use of a simple box-car filter with duration roughly equal to that of the M2 tidal cycle. Since the model output was only stored every 10 min, it was found that double use of this filter was required to remove an M2 oscillation in the residual still present on first use of the filter due to the slight mismatch between the box-car duration and the M2 period. Double use of the filter yielded the outcome in Fig. 8a and b which shows a southerly residual flow that roughly doubles in strength from neap to spring tide from 334 m\(^3\) s\(^{-1}\) to 686 m\(^3\) s\(^{-1}\) (see Table 2). This agrees quite well with the neap-to-spring variation of 330 to 800 m\(^3\) s\(^{-1}\) estimated by Simpson et al. (1971) and is consistent with the subsequent flux estimate made by Campbell et al. (1998). The overall mean flux was obtained by applying a further 14-day box-car filter, which yields a settled residual after about 14 days until the end of the simulation at 20 days. The mean flux was thus estimated for the SN-cycle with variable \( k_s \), as \(-525\) m\(^3\) s\(^{-1}\).

Equivalent results for the SN-cycle obtained with the various fixed \( k_s \) values are shown in Fig. 8b from which the results shown in Fig. 9 have been extracted to highlight the residual fluxes at peak springs and neaps together with the overall SN-mean flux. The spring tide residual velocity increases significantly in magnitude as the constant \( k_s \) value increases. In contrast, the magnitude of the neap tide residual, while varying over a smaller range as expected, decreases to a minimum when \( k_s = 0.3 \) m. The overall SN-mean residual was calculated as described above and is shown in Table 2 in comparison with the average of the spring and neap residual values. While the (negative) SN-mean value increases in magnitude as the constant \( k_s \) value increases, it remains roughly constant in the centre of the \( k_s \) range shown. In order to recover the variable roughness outcomes (dashed lines) using a fixed value of \( k_s \), the required values are approximately: Spring residual, \( k_s \approx 0.27 \) m; Neap residual, \( k_s \approx 0.17 \) m; Overall SN residual, \( k_s \approx 0.085 \) m.

Each of these values is significantly smaller than the mid-channel values of roughness shown earlier, suggesting that use of a mid-channel value (Fig. 6) as the overall fixed \( k_s \) value would underestimate peak flow rates, while overstating the residual flow. The values of \( k_s \) themselves are in reasonable agreement with those obtained by Rippeth et al. (2002) based on the logarithmic profile method, but are substantially larger than their estimate determined by the turbulence ‘variance method’ (see Section 3).

For the 50-hour M2-alone runs the residual flow computations involved a similar process of double-filtering to that used for the SN-cycle. With spatially-temporally varying \( k_s \), the residual flux for M2-alone was \(-491\) m\(^3\) s\(^{-1}\), which is about 7% smaller in magnitude than the value for the SN-cycle. Again M2-alone results were obtained for comparison using a range of fixed constant \( k_s \) values. For variable \( k_s \), the peak northward and southward fluxes across the transect were 2900 m\(^3\) s\(^{-1}\) and \(-4682\) m\(^3\) s\(^{-1}\), reflecting the dominance of the flow towards the southwest. Fig. 10a shows a comparison between these values and the results obtained for M2-alone with fixed constant \( k_s \) values. Both the northward and southward peak fluxes decrease in magnitude significantly as \( k_s \) increases. In contrast, the relatively small residual velocity, shown in detail in Fig. 10b, increases in magnitude as \( k_s \) increases, as found in Figs. 8b and 9 for the SN-cycle. In order to recover the variable roughness outcomes (dashed lines), the required fixed values of \( k_s \) are approximately: Peak northeast flow, \( k_s \approx 0.17 \) m; Peak southwest flow, \( k_s \approx 0.25 \) m; Residual flow, \( k_s \approx 0.085 \) m.

Again, each value is significantly smaller than the mid-channel values of dune roughness shown in Fig. 6.

5.3. ‘Equivalent’ M2 tide and overall predictions of \( k_s \)

Comparing the outcomes for the residual flow in the case of variable \( k_s \) (Table 2), the overall mean values for the SN-cycle (\(-525.4\) m\(^3\) s\(^{-1}\)) and M2-alone (\(-490.8\) m\(^3\) s\(^{-1}\)) differ by about 7% due probably to the enhanced contribution to the flux from spring tides compared with neaps. In order to match the SN-cycle value, an enhanced ‘equivalent’
$M_2$-tide can be defined. This has been determined by re-running the $M_2$-alone case with variable roughness using an enhanced $M_2$-amplitude. In particular, the time-varying tidal heights have been rescaled uniformly at both model open sea boundaries from which it has been found that the SN-residual can be recovered by an ‘equivalent $M_2$’ tide of amplitude enhanced by 7.3%. It would seem from this that the residual flow in the Strait is related approximately linearly to the $M_2$ amplitude. This enhancement is qualitatively similar to the ‘morphological tide’ determined by Latteux (1995) as the best single equivalent tide to represent sediment transport during a full SN-cycle. This morphological tide lies between mean and spring tidal conditions, due to the strong nonlinear relationship between sediment transport and flow velocity that causes the bed morphology to be related more to spring tides, hence requiring an enhancement of the mean $M_2$ amplitude. Similar considerations evidently apply in the definition of the present ‘equivalent $M_2$’ tide.

Finally, it may be noted that, if the predicted variable $k_s$ values are spatially averaged within the Menai Strait, between locations A and C (Fig. 1a), the value obtained at the end (after 50 h) of the $M_2$-alone simulation is $k_s \approx 0.15$ m. This is about a quarter of the peak value of $k_s$ typically resulting from the dunes in mid-channel (Figs. 3, 6). The spatially-averaged value for the entire model domain is about $k_s \approx 0.20$ m. These spatially-averaged $k_s$ values are somewhat smaller than the range of $k_s$ values of 0.2–0.3 m which provides a reasonable representation of both the instantaneous peak fluxes and also the residual flux for $M_2$ alone. This points to the dynamics in the Strait being more influenced by the larger roughness encountered by the stronger flow in mid-channel, than by the shallower areas where smaller $k_s$ values are predicted. In summary, the effective roughness of the seabed in the Menai Strait is <50% of the maximum local roughness due to the dunes in mid-channel, but more than the spatially mean predicted roughness in the Strait as a whole by about 50%.

### 6. Discussion: $k_s$ observed versus predicted

As illustrated in Fig. 3, Van Rijn’s (2007) predictions for $k_s$ are dominated in mid-channel by the contribution from dunes, with smaller contributions arising from mega-ripples and small-scale ripples (Fig. A1). The predicted overall roughness, augmented by the new ‘history effect’, gives rise to realistic water fluxes. However while dunes are predicted to form for the velocities, grain sizes and water depths occurring in the Menai Strait, a question arises as to whether these dunes formed in tidal flow achieve the wavelengths and heights that would occur on an extended timescale in an equivalent riverine setting involving steady flow for which Van Rijn’s (2007) formulation may be most appropriate. In particular, is Van Rijn’s (2007) suggestion (see Appendix A, Section A1) that the
contribution $k_d$ of dunes is ‘roughly on the order of half the bedform height’ borne out by observations of the dunes on site. Observations of dunes in the Menai Strait were made by McAleese (2012) at a mid-channel location in the vicinity of Traeth Gwyllt (Fig. 1a), where the water depth, bed sediment size and flow conditions were closely similar to those considered in Section 5. The main aim of these observations was to assess whether any significant variations occurred in dune length, height or asymmetry on intra-tidal timescales. Multibeam echo sounder data were collected on 19th April 2012 by Bangor University. The system was successful in measuring changes in dune heights of the order of 0.02 to 0.04 m and the overall vertical and horizontal resolution achieved was ±0.02 m, with sampling carried out at a nominal horizontal interval along the bed profiles of 0.01 m. Four along-channel profiles (P1–P4) (Fig. 11a) of length 110–120 m were interrogated at nominally 40-minute intervals for a period of 5.5 h on a falling ebb tidal flow towards the southwest with water depths decreasing from about 15 m to 11 m.

The dune shapes observed along each of the mid-channel profiles were regular and did not exhibit any systematic change in height and wavelength during the course of the ebb tide. The mean heights and wavelengths of the dunes ($\eta_d$, $\lambda_d$) averaged for each along-channel profile P1 to P4 were (0.47 m, 7.4 m), (0.32 m, 6.1 m), (0.21 m, 5.1 m) and (0.34 m, 5.9 m), respectively (see Table 3). However, a complete reorientation of the dune crest symmetry, from northeast facing to southwest facing, was measured over the course of the survey period. This was accompanied by the migration of the dune crests in the southwest direction by distances ranging from 0.4 m to 5.5 m. According to the broad regimes defined for tidal bedforms by Van Rijn (1993) (see Section 2) the present observations of ($\eta_d$, $\lambda_d$), with a representative sediment median grain size of 0.25 mm, would appear to be of ‘mega-ripples’. However, Van Rijn’s (2007) scheme used here predicts the bed roughness to be dominated by the dune contribution $k_d$.

With the horizontal length of the dune lee slope taken as $\lambda_1$ and the dune height as $\eta_1$, values of the dune symmetry ratio $\lambda_1/\eta_1$ decreased from an overall mean value of 11.5 to 7.7 during the 5.5 h survey period (see Table 3). The corresponding change in Van Rijn’s form factor $\gamma_d$ (see Section 2) was from an overall mean value of 0.17 to 0.51. At the onset of the ebb tide, the low value of $\gamma_d$ resulted from the shallow lee slope left by the previous flood tide (cf. Rippeth et al., 2002). As the dunes reoriented on the ebb tide, the lee slope steepened and $\gamma_d$ increased to its final value of about 0.51. The mean value of $\gamma_d$ of 0.30 during this process was about one half of Van Rijn’s (1993) expected value of 0.70 for dunes in field conditions. This provides a plausible explanation for the factor of 2 difference between the modelled $k_d$ values which here are roughly equal to the observed dune heights and Van Rijn’s expectation that $k_d$ should be ‘roughly on the order of half the bedform height’ (see Appendix A).

The discrepancy arising from this assumption between the observations (plotted with $k_d = 0.5\eta_1$) and model predictions of the dune roughness made with and without a ‘history effect’ applied are shown in Fig. 11b for profile P1. Values of the development rate parameter $\gamma_d$ were taken as $0.5 \times 10^6$ and $2.5 \times 10^6$. Slack water at high and low tide occurred on site at hours 32.6 and 39.4, respectively. The model runs were carried out with tidal inputs at either end of the Strait covering the survey period, with a run duration of 2 days and a representative bed grain size of 0.25 mm. The use here of a mixture of grain sizes (cf. Section 4.2), leading to some spatial variations in $d_{50}$, might have given rise to some minor changes in the results in Fig. 11b. The predicted surface elevations were shown to match closely the tidal curve measured on site. In the case of profile P1 for which the mean value of $\gamma_d$ was 0.47, the implied $k_d$ value is about 0.57 times the modelled value averaged over the survey period. The corresponding ratios for profiles P2, P3 and P4 were 0.41, 0.25 and 0.42, which can be seen in Table 3 to be related to the respective values of $\lambda_1/\eta_1$. The model results in Fig. 11b show how the ‘history effect’ suppresses the large temporal variations in $k_d$ predicted at slack water both before and after the survey period. As expected, the larger is the value of the maximum development rate parameter $\gamma_d$ (Appendix A) the more strongly damped is the temporal variation in $k_d$.

If it is accepted from the earlier water flux computations that Van Rijn’s (2007) roughness predictor gives a satisfactory representation of the bed friction, then it seems reasonable to suggest that in a reversing tidal flow in which dunes attempt to form, but are not able to develop fully, Van Rijn’s predictor remains applicable but with the important caveat that $k_d$ is roughly on the order of the bedform height, rather than half that height. This conclusion could possibly be refined through consideration of the role of the parameter $\lambda_1/\eta_1$. Values of $\lambda_1/\eta_1$ in the approximate range 8 to 12 found in the present tidal setting are larger than would be expected in a riverine setting (for which $\lambda_1/\eta_1 \approx 6$ corresponds to Van Rijn’s suggested value of $\gamma_d \approx 0.7$). This difference is associated with the more symmetrical bedforms that occur in reversing flow than in a comparable steady flow.

In summary, compared with dunes in river flow, the dunes observed by McAleese (2012) had (i) shorter wavelength $\approx h$; (ii) more symmetrical profiles with less steep lee slopes and (iii) $k_d$ on the order of the bedform height in most cases, rather than half the bedform height as suggested by Van Rijn (2007). However, the dunes being more numerous due to their smaller wavelength, they
still exert significant frictional influence which seems to be well represented by Van Rijn’s (2007) formulation subject to the caveat that in reversing tidal flow $k_s/\eta_d$ is $\approx 1$. The more symmetrical, more numerous, bedforms promote the turbulence intensity in the reversing flow and hence give rise to larger $k_s$ values than would be expected in a river.

Finally, as noted in Section 4.2, the ‘history effect’ on the dune roughness $k_{\text{red}}$ inhibits intra-tidal changes in $k_s$, but still allows variations on a SN-timescale, though only limited observational evidence exists to support this latter supposition. Shaarani (2013) made further observations of dunes along the same survey profile lines P1 to P4 focussing on differences between spring and neap conditions. These observations suggested that dune lengths and heights were greater by 18% and 41%, respectively, on spring compared with neap tides. Such changes would appear to be consistent both with the observations reported by Rippeth et al. (2002) and also with the present prediction of $k_s$ variations on this time scale, though more systematic observations are needed to tie down the value of development rate parameter $\gamma_s$ (see Appendix A). Further, the present ‘history effect’ takes no account of the variation of parameter $\lambda_s/\eta_d$ on intra-tidal time scales, which could be incorporated in the formulation.

7. Conclusions

The use of a spatially and temporally variable, predicted, bed roughness $k_s$ in coastal and estuarine morphodynamic modelling provides greater realism than the use of a constant, tuned value of $k_s$. This has been assessed by comparing Telemac modelling outcomes for a tidal channel (Menai Strait, N. Wales) obtained with variable $k_s$ based on Van Rijn’s (2007) prediction scheme, involving dunes, mega-ripples and small-scale ripples, with outcomes obtained for constant $k_s$ values. Van Rijn’s scheme has been accompanied by a new ‘history effect’ that prevents the dune roughness contribution from making unrealistically rapid, intra-tidal, changes.

The bed roughness scheme, available in the Telemac morphodynamic module Sisyphe, has been assessed by taking as a diagnostic the magnitude of the residual flow that occurs in the channel due to differences in tidal range and phase at its two open ends. Van Rijn’s

Fig. 7. (a) Peak northward (positive) and southward (negative) flows across the Flux transect during the final cycle of the $M_2$-alone simulation. The $v$-velocity profiles obtained with variable $k_s$ are shown as full lines in the respective directions. The results using fixed $k_s$ values are shown by the dashed curves which correspond to those in Fig. 6. (b) Water depth (m) across the Flux transect relative to Mean Sea Level (MSL) together with High Water (HW) and Low Water (LW) elevations during the $M_2$ cycle (dashed lines).
scheme predicts dune roughness in mid-channel tending to small-scale ripple roughness at the channel margins, as observed. The scheme also provides an accurate prediction of the residual flow that agrees with previous estimates and observations made in the Menai Strait. Compared with the use of a more traditional, constant $k_s$ value to represent bottom friction throughout the model domain, variable $k_s$ introduces subtle differences in the model outcomes, for example by slowing down the current speed in mid-channel while allowing greater speeds at the channel margins. Such differences, even if relatively small in magnitude, have the potential to alter long-term morphodynamic outcomes.

For a mean SN-cycle with variable $k_s$, the mean residual flow through the Menai Strait is predicted to be $525 \text{ m}^3 \text{s}^{-1}$ towards the southwest, varying between $334 \text{ m}^3 \text{s}^{-1}$ on neaps and $686 \text{ m}^3 \text{s}^{-1}$ on springs, consistent with observations. With a constant $k_s$ value imposed, this residual flow is recovered approximately using values of $k_s$ in the range $0.2 \text{ m}$ to $0.3 \text{ m}$. Such values are significantly smaller than the peak values of $k_s$ predicted in the deeper water of the main channel, suggesting that the use of a mid-channel $k_s$ value as an overall choice for the system may not be appropriate resulting, for this system at least, in an overstating of the residual flow. In fact, if the predicted spatially varying $k_s$ is averaged throughout the Menai Strait this results in a value of about $0.15 \text{ m}$. So the effective roughness of the seabed ($k_s \approx 0.2 – 0.3 \text{ m}$) is less than half the maximum local roughness due to the dunes predicted in mid-channel, but more than the spatially mean predicted $k_s$ value in the Strait as a whole by about 50%.

The tidal simulations carried out for $M_2$-alone using variable $k_s$ produced a somewhat smaller (by 7%) residual flow of $491 \text{ m}^3 \text{s}^{-1}$ towards the southwest. The use of an ‘equivalent $M_2$’ tide of amplitude enhanced by 7.3% reconciled these estimates, suggesting that the residual flow magnitude is related approximately linearly to the $M_2$ tidal amplitude. Further, the unadjusted $M_2$-alone values can be recovered approximately by using constant $k_s$ values in the range $0.1 \text{ m}$ to $0.3 \text{ m}$, an outcome similar to that found for the SN-cycle.

Table 2

Residual flow in the Menai Strait calculated for the SN-cycle using variable predicted $k_s$ and constant fixed $k_s$ values. The negative values signify flow towards the southwest end of the Strait, with larger flow magnitudes occurring on springs than neaps. The average of the spring and neap values is shown in brackets for comparison with the overall SN-mean value.

<table>
<thead>
<tr>
<th>Residual Flow (m$^3$ s$^{-1}$)</th>
<th>Variable $k_s$</th>
<th>$k_s = 0.05 \text{ m}$</th>
<th>$k_s = 0.15 \text{ m}$</th>
<th>$k_s = 0.30 \text{ m}$</th>
<th>$k_s = 0.45 \text{ m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring max</td>
<td>$-685.9$</td>
<td>$-638.9$</td>
<td>$-669.2$</td>
<td>$-689.5$</td>
<td>$-731.1$</td>
</tr>
<tr>
<td>Neap min</td>
<td>$-334.3$</td>
<td>$-386.0$</td>
<td>$-372.6$</td>
<td>$-343.4$</td>
<td>$-384.0$</td>
</tr>
<tr>
<td>SN mean</td>
<td>$-525.4$</td>
<td>$-501.3$</td>
<td>$-528.5$</td>
<td>$-529.6$</td>
<td>$-577.1$</td>
</tr>
<tr>
<td>(av. max &amp; min)</td>
<td>$(-510.1)$</td>
<td>$(-512.5)$</td>
<td>$(-520.9)$</td>
<td>$(-516.5)$</td>
<td>$(-568.6)$</td>
</tr>
</tbody>
</table>

Fig. 8. (a) Predicted instantaneous water flow though the Flux transect during a SN-cycle with variable roughness $k_s$. Positive values of flux are northward. Double box-car filtering of the instantaneous fluxes gives rise to the averaged residual outcome. (b) The slowly-changing residual water flux through the SN-cycle is shown in more detail, together with the corresponding fluxes for four fixed values of $k_s$.

(2007) scheme predicts dune roughness in mid-channel tending to small-scale ripple roughness at the channel margins, as observed. The scheme also provides an accurate prediction of the residual flow that agrees with previous estimates and observations made in the Menai Strait. Compared with the use of a more traditional, constant $k_s$ value to represent bottom friction throughout the model domain, variable $k_s$ introduces subtle differences in the model outcomes, for example by slowing down the current speed in mid-channel while allowing greater speeds at the channel margins. Such differences, even if relatively small in magnitude, have the potential to alter long-term morphodynamic outcomes.

For a mean SN-cycle with variable $k_s$, the mean residual flow through the Menai Strait is predicted to be $525 \text{ m}^3 \text{s}^{-1}$ towards the southwest, varying between $334 \text{ m}^3 \text{s}^{-1}$ on neaps and $686 \text{ m}^3 \text{s}^{-1}$ on springs, consistent with observations. With a constant $k_s$ value imposed, this residual flow is recovered approximately using values of $k_s$ in the range $0.2 \text{ m}$ to $0.3 \text{ m}$. Such values are significantly smaller than the peak values of $k_s$ predicted in the deeper water of the main channel, suggesting that the use of a mid-channel $k_s$ value as an overall choice for the system may not be appropriate resulting, for this system at least, in an overstating of the residual flow. In fact, if the predicted spatially varying $k_s$ is averaged throughout the Menai Strait this results in a value of about $0.15 \text{ m}$. So the effective roughness of the seabed ($k_s \approx 0.2 – 0.3 \text{ m}$) is less than half the maximum local roughness due to the dunes predicted in mid-channel, but more than the spatially mean predicted $k_s$ value in the Strait as a whole by about 50%.

While Van Rijn’s (2007) bed roughness scheme gives rise to the occurrence of dunes in the present tidal channel, the bedforms observed on site are smaller in height and length than predicted. This raises important questions about the applicability of roughness schemes developed for steady, riverine conditions in tidal conditions where repeated flow reversals prevent equilibrium dunes from forming. It may be concluded from the present study that, while Van Rijn’s (2007) scheme provides a realistic estimation of the roughness $k_s$, the relationship between $k_s$ and the dunes on site differs from that occurring in a steady flow. In particular, $k_{sd}$ appears to be roughly on the order of the bedform height in reversing flow, rather than half that height as expected in steady flow. This relationship arises from
the dunes in the tidal setting being smaller in wavelength, but therefore more numerous, and at the same time more upright and symmetrical than dunes in a river. The dune behavior in tidal flow seems also to involve significant intra-tidal changes in bedform profile asymmetry that alter the roughness and that could, potentially, be included to improve future models.

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Wales, the Welsh European Funding Office, and the European Regional Development Fund Convergence Programme. This project was undertaken in collaboration with the National Grid, which organized the inter-tidal LiDAR survey in 2013. The MBES survey in 2012 was conducted by several staff at Bangor University, including Michael Roberts, Guy Walker-Springett and James Bennell, who processed the data. The dune observations off Traeth Gwyllt were made by Gail McAleese, who also processed the data under the supervision of James Bennell.

All of the above are thanked, as are Jaco Baas and also the reviewers of the paper for their helpful comments.

Appendix A. Bed roughness prediction, ‘history effect’ and modelling methodology

The formulation of Van Rijn (2007) was used to determine the roughness of sub-grid-scale bed features. The roughness concept may

Table 3
Dune observations made by McAleese (2012) in the vicinity of Traeth Gwyllt ($\eta_d =$ dune height, $\lambda_d =$ dune length, $\lambda_1 =$ horizontal length of dune lee slope, $\gamma_d =$ form factor).

<table>
<thead>
<tr>
<th>Profile</th>
<th>$\eta_d$ (m) mean</th>
<th>$\lambda_d$ (m) mean</th>
<th>$\lambda_1$ (m) initial</th>
<th>$\lambda_1$ (m) final</th>
<th>$\lambda_1/\lambda_d$ initial</th>
<th>$\lambda_1/\lambda_d$ final</th>
<th>$\lambda_1/\lambda_d$ mean</th>
<th>$\gamma_d$ initial</th>
<th>$\gamma_d$ final</th>
<th>$\gamma_d$ mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0.47</td>
<td>7.4</td>
<td>4.4</td>
<td>3.0</td>
<td>9.4</td>
<td>6.3</td>
<td>7.8</td>
<td>0.29</td>
<td>0.70</td>
<td>0.47</td>
</tr>
<tr>
<td>P2</td>
<td>0.32</td>
<td>6.1</td>
<td>3.6</td>
<td>2.4</td>
<td>11.5</td>
<td>7.7</td>
<td>9.6</td>
<td>0.15</td>
<td>0.48</td>
<td>0.26</td>
</tr>
<tr>
<td>P3</td>
<td>0.21</td>
<td>5.1</td>
<td>3.1</td>
<td>2.0</td>
<td>14.6</td>
<td>5.7</td>
<td>12.1</td>
<td>0.06</td>
<td>0.25</td>
<td>0.12</td>
</tr>
<tr>
<td>P4</td>
<td>0.34</td>
<td>5.9</td>
<td>3.5</td>
<td>2.4</td>
<td>10.6</td>
<td>7.0</td>
<td>8.8</td>
<td>0.19</td>
<td>0.59</td>
<td>0.35</td>
</tr>
</tbody>
</table>
be traced back to the equivalent sand roughness height \( (k_s) \) of Nikuradse which, in the case of a movable bed of sediment, is subdivided into the grain roughness \( (k_{s,grain}) \) arising from skin friction forces and form roughness \( (k_{sr}) \) generated by pressure forces acting on the bedforms.

A.1. Quasi-steady bed roughness prediction

Van Rijn’s (2007) formulation for the bed roughness contributions arising from small-scale ripples \( k_{sr} \), mega-ripples \( k_{smr} \) and dunes \( k_{sd} \) depends upon a single parameter, namely the mobility number, defined in the absence of waves as:

\[
\psi = U_c^2/((s-1)gd)
\]  

where \( U_c \) is the depth-mean current speed; \( s = \rho_s / \rho \) is the relative sediment density (\( \rho_s \) and \( \rho \) are the densities of sediment and water respectively); \( g \) is the acceleration due to gravity; and \( d = d_{50} \) is the median grain diameter. An example of results from the predictor is given in Fig. A1. [If waves are present \( U_c^2 \) is replaced by \( (U_c^2 + U_w^2) \) where \( U_w \) is the near-bed wave velocity amplitude.]

The local instantaneous roughness contributions were computed in Sisyphe, as follows:

i) the roughness of movable small-scale ripples \( (k_{sr}) \) is assumed to be equal approximately to the ripple height. The ripples are assumed to be ‘fully developed’ and so growth and history effects are not included. The ‘smooth’ formula proposed by Van Rijn (2007) to represent \( k_{sr} \) over the full \( \psi \)-range is as follows:

\[
k_{sr} = f_{cs}d_{50}[85-65\exp(0.015(\psi-150))] \]

where either the coefficient \( f_{cs} = (0.25d_{gravel}/d_{50})^{1.5} \) with \( d_{gravel} = 0.002 \) m, or \( f_{cs} = 1 \) for \( d_{50} \leq 0.25d_{gravel} \). For fine sediment of silt size a separate expression is used:

\[
k_{sr} = 20d_{silt} \text{ for } d_{50} \leq d_{silt} \text{ where } d_{silt} = 0.000032 \text{ m} \]

The predicted ripple roughness is maximum in the lower current regime, while the ripples are suppressed at high flow stages, ultimately disappearing in the upper current regime \( (\psi \geq 250) \).

ii) mega-ripples \( (k_{smr}) \) have a different behavior in that they tend first to increase in height, and then decrease, for increasing \( \psi \). The wavelengths of mega-ripples are of the order of, but no greater than, the mean water depth \( (h) \). Van Rijn (2007) indicates that the roughness \( (k_{smr}) \) is ‘roughly on the order of half the bedform height’. Thus the dominance of small-scale ripples at low wave-current mobility is replaced, in part, by the presence of mega-ripples at higher mobility. Van Rijn’s (2007) direct parameterisation of \( k_{smr} \) over the full \( \psi \)-range is as follows:

\[
k_{smr} = 0.00002f_wh[1-\exp\left(-0.05(\psi)\right)](500-\psi) \]  

where the factor \( f_w = d_{50}/1.5d_{sand} \) where \( d_{sand} = 0.000062 \) m, or \( f_w = 1 \) for \( d_{50} \leq 1.5d_{sand} \). At high flow stages the following expressions are used:

\[
k_{smr} = 0.02m \text{ for } \psi > 550 \text{ and } d_{50} \geq 1.5d_{sand} \]

\[
k_{smr} = 200d_{50} \text{ for } \psi > 550 \text{ and } d_{50} < 1.5d_{sand} \]

For fine sediment of silt size the mega-ripples are inhibited:

\[
k_{smr} = 0 \text{ for } d_{silt} \leq d_{50} \]

iii) dunes \( (k_{sd}) \) have different characteristics compared with mega-ripples, mainly by being strongly dependent upon the water depth \( (h) \) and having wavelengths larger than the depth \( (h) \) in the range \( 3h \text{ to } 15h \). The dunes achieve significant heights and, like mega-ripples, induce form roughness. Also like mega-ripples they first increase, and then decrease, in height for increasing mobility. As for mega-ripples, Van Rijn (2007) indicates that the physical bedform roughness \( (k_{sd}) \) of dunes is ‘roughly on the order of half the bedform height’. The formula proposed by Van Rijn (2007) for the full \( \psi \)-range is:

\[
k_{sd} = 0.00008f_{sd}h[1-\exp\left(-0.02(\psi)\right)](600-\psi) \]

where the factor \( f_{sd} \) is as defined earlier. For high mobility and for fine-grained sediment, the dunes are inhibited:

\[
k_{sd} = 0 \text{ for } \psi > 600 \text{ and for } d_{50} \leq d_{silt} \]
The above formulation allows maximum \( k_{s,d} \) values to increase as \( h \) increases up to a maximum allowed value of \( k_{s,d} = 1 \) m. The dune roughness typically far exceeds the contributions made to the total roughness by mega-ripples and small-scale ripples.

iv) The overall roughness due to the respective sub-grid-scale features, which may be superimposed upon one another in the coastal domain, is obtained by quadratic summation:

\[
k_{s,\text{TOT}} = \left[ k_{s,\text{gr}}^2 + k_{s,\text{fl}}^2 + k_{s,\text{ad}}^2 \right]^{\frac{1}{2}}
\]

(A.9)

This quadratic expression, rather than a simple summation, is acknowledged by Van Rijn (2007) to be an intuitive, engineering approach. It is possibly justified for situations where the bedform crest lines are not aligned, such that flows directed locally and instantaneously, along crests (and troughs) of one or other of the bedform components will not encounter the full roughness of that component but, instead, a sharply reduced value (cf. Soulsby, 1990). Thus a simple addition of the three roughness components to determine \( k_{s,\text{TOT}} \) might lead to an overestimate.

The total roughness is calculated in Sisyphe at each point of the computational domain, using Eqs. (A.1)–(A.9); to the quadratic summation in Eq. (A.9) is added a granular, skin friction, component \( k_f \). The total roughness is passed back from Sisyphe to Telemac2D. The flow calculated in Telemac2D adjusts to the evolving roughness and the velocity field is passed back in turn to Sisyphe. This interaction between the modules has been robust and stable in all simulations carried out to date. The sediment transport calculations in Sisyphe use only the small-scale ripple roughness \( k_{sr} \) (Eqs. (A.2)–(A.3)) together with skin friction effects \( (k_f^*) \) that are worked out here according to Bijker’s (1992) transport formulation (see Davies and Villaret, 2002).

A.2. History effect for dune roughness

The quasi-steady application of the earlier formulae through a tidal cycle implies an equilibrium response of the bed roughness to changes in the flow conditions. In practice, such a rapid response may be unrealistic due to the time required for bedforms to grow or to be destroyed. In other words, there is a ‘history effect’ in the bedform response. Here only the dune roughness component is considered, though the same principle and methodology may be applicable also to mega-ripples. The aim of the new ‘history effect’ is to inhibit the rate of bedform growth/destruction in a realistic manner.

The time scale of dune evolution will depend upon the strength of the flow \( U \), the sediment grain size \( d \) (=\( d_{50} \)), the mean water depth \( h \), and possibly also the acceleration due to gravity \( g \), allowing a Froude number dependence. So, for example, the time scale \( T_d \) for the growth of a dune might be governed by \( T_d = d^2 / g U \) such that the larger the grains the longer the time scale, and the larger the velocity the shorter the time scale.

Suppose that it takes, for example, about 6 h \( \approx 2 \times 10^4 \) s for ‘equilibrium’, i.e. fully developed, dunes to form in a current of strength 1 m s\(^{-1}\) for a grain diameter of \( d = 0.2 \) mm = \( 2 \times 10^{-4} \) m, then \( d / U = 2 \times 10^{-4} \) s and \( T_d = \sqrt{g / \gamma_h} = 0(10^6) \). Such a time scale provides the simplest possible basis upon which to define a lag effect, or ‘history effect’, for dune growth or erosion. If the quasi-steady approach to the calculation of \( k_f \) implies a dune growth/destruction rate that is faster than which can possibly be attained according to the time scale \( T_d \), then the growth/destruction rate of the dunes should be inhibited. It is assumed in the algorithm applied here that there is no distinction between inhibition on growth and that on destruction.

New procedures have been introduced into Sisyphe to implement the dune ‘history effect’. Suppose that the existing bed roughness at a typical grid node within the domain is denoted by \( k_{s,\text{now}} \). At the next iteration (after interval \( \Delta t \)) the model computes a new, quasi-steady, set of equilibrium values of the bed roughness \( k_{s,\text{eq}} \) say, based on the mobility number \( \psi \) at each grid node. For the bedforms to develop at the rate implied by this equilibrium value, the bed roughness \( k_f \) needs to change at the ‘target’ rate:

\[
\frac{\Delta k_f}{\Delta t} = \frac{k_{s,\text{eq}} - k_{s,\text{now}}}{\Delta t}
\]

which, according to Van Rijn’s (2007) rule (Section 6), implies a ‘target’ rate of change of dune height \( \gamma_h \) of \( 2 \times \Delta k_f / \Delta t \). The question that arises is whether this target rate is greater than the maximum allowable growth rate for the existing flow state (i.e. stage of the tide), namely:

\[
\frac{\Delta \gamma_h}{\Delta t} = \frac{k_{s,\text{eq}} - k_{s,\text{now}}}{\Delta t}
\]

(A.10)

The new algorithm limits the dune growth or destruction rate to be no greater than the maximum allowable rate, at that location, and at that time.

A.3. Implementation of the ‘history effect’ in Sisyphe

Consider the case of a growing dune. If it takes time \( T_d \) for a dune of wavelength \( \lambda_d \) to grow to equilibrium (height \( \gamma_h \equiv \gamma_h\text{eq} \)) then the assumed maximum growth rate for dune height \( \gamma_h \) is \( \gamma_h \text{eq} / T_d \). If the target growth rate \( \gamma_h / T_d \) is compared with \( \gamma_h \text{eq} / T_d \) then the following ‘history effect’ is imposed at that location in the domain:

\[
k_{s,t+1} = \begin{cases} k_{s,\text{eq}} & \text{if } \frac{\Delta k_f}{\Delta t} \leq \frac{\gamma_h \text{eq}}{T_d} \\ k_{s,t} + (k_{s,\text{eq}} - k_{s,t}) \times \left( \frac{\gamma_h \text{eq} / T_d}{\frac{\Delta k_f}{\Delta t}} \right)^R & \text{if } \frac{\Delta k_f}{\Delta t} > \frac{\gamma_h \text{eq}}{T_d} \end{cases}
\]

(A.11a) or

\[
k_{s,t+1} = k_{s,t} + (k_{s,\text{eq}} - k_{s,t}) \times \left( \frac{\gamma_h \text{eq} / T_d}{\frac{\Delta k_f}{\Delta t}} \right)^R
\]

(A.11b)

with the rate given by power \( R \) set to 1 as a first guess, which seems applicable if \( \gamma_h \text{eq} / T_d \) is specified as a function of the flow speed. A schematic of inhibited roughness growth is depicted in Fig. A2. An equivalent procedure has been programmed for decaying dunes.

The implementation requires knowledge of the roughness values \( k_{s,t} \) at the old time step. At each iteration of Sisyphe the newly calculated roughness values have been retained for further use. At the next iteration of Sisyphe, these values \( k_{s,\text{now}} \) are used to determine the target dune growth rate \( \Delta k_f / \Delta t = k_{s,\text{eq}} - k_{s,\text{now}} \) and, hence, the history effect in the roughness (via Eqs. (A.11a) and (A.11b)). The procedure appears to be numerically robust and stable, and it is also physically meaningful. What is still

![Fig. A2. Scheme for 'history effect' in the case of a growing dune showing the instantaneous target \( k_{s,eq} \) at the next time step, together with the imposed value \( k_f \), implying an inhibition in the rate of change of the roughness.](image-url)
needed is additional site information with which to tune the calibration constant $\gamma_s$ (and also, subsequently, $R$), and to determine whether growth and decay can really be characterized by the same time scale $T_D$ or whether two different time scales are needed.

References


