1DV-MODEL OF SAND TRANSPORT BY WAVES AND CURRENTS IN THE RIPPLED BED REGIME

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Abstract: A new two-layer, one-dimensional vertical (1DV) model of wave-induced flow and sand transport above a rippled bed is presented. Above steep ripples, momentum transfer and the associated sediment dynamics are dominated in the near-bed layer by coherent motions, specifically by the process of vortex formation and the shedding of these vortices at flow reversal. This process is represented by means of a strongly time-varying eddy viscosity, with peaks occurring just ahead of flow reversal. Above this near-bed layer, the flow is represented by a conventional One-equation turbulence-closure scheme. The model results are compared with data obtained in a large-scale flume. Here suspended sand concentration profiles, and the bed form dimensions, were measured with considerable accuracy using acoustic techniques. The model results are compared with the measured mean concentration profiles, and a multiple grain size approach is used to predict the observed height-variation in the suspended sediment grain size. In addition, a comparison is presented with field data for suspended sand concentrations, obtained in wave-current flow, which demonstrates the improved agreement achieved by the new rippled bed model in comparison with a standard flat-bed modelling approach.

INTRODUCTION

Fundamentally different physical processes determine sediment transport rates above plane and rippled sand beds. Plane beds occur in the nearshore zone beneath large (e.g. storm) waves, while rippled beds are generated by less active wave and current conditions and occur extensively in deeper water further offshore. Both bed
types have considerable practical importance, not least in relation to morphological modelling, and both require appropriate modelling approaches.

Above plane beds in oscillatory flow, momentum transfer occurs primarily by turbulent diffusion and may, together with the associated sediment transport, be modelled using conventional turbulence-closure, numerical, schemes. Such schemes may include detailed treatment of the very near-bed, sheet flow layer (see, for example, Malarkey et al., 2002). In contrast, above rippled beds, momentum transfer and the associated sediment dynamics are dominated in the near-bed layer by coherent motions, specifically by the process of vortex formation above ripple lee slopes, and the shedding of these vortices at flow reversal. Above steep, long-crested ripples, with height to wavelength ratio greater than about 0.12, this well-organised, ‘convective’ process of vortex formation and shedding is highly effective in entraining sand into suspension. The rough turbulent oscillatory boundary layer above rippled beds is considerably thicker than that above plane beds, the boundary layer consisting of a lower layer dominated by the vortex shedding process and an upper layer in which the coherent motions break down and are replaced by random turbulence. This leads to the entrainment of sediment into suspension to considerably greater heights than above plane beds. In combined wave and current flow above ripples, the outer part of the boundary layer structure referred to above merges with the turbulent ‘current’ boundary layer, into which sediment can be entrained into suspension to still greater heights.

Several modelling studies of two-dimensional horizontal-vertical (2DHV) oscillatory flow above rippled beds have sought to represent the formation and shedding of vortices, and the subsequent trajectories of the (decaying) vortices. Some of these studies have also investigated integrated quantities such as the wave friction factor (drag coefficient), as well as time-mean residual flow structures above ripples. For example, numerical solutions of the governing vorticity equation have been developed by Sleath (1973) and Blondeaux and Vittori (1991). Longuet-Higgins (1981), and also Block et al. (1994), proposed essentially analytical discrete-vortex models of the oscillating flow above steep ripples, while Hansen et al. (1994), Perrier (1996) and, more recently, Malarkey (2001) developed numerical discrete-vortex models. Turbulence-closure models have also been used; for example, Perrier (1996) developed a Reynolds stress closure model to investigate hydrodynamic and sediment transport processes, and Andersen and Fredsøe (1999) used a k-ω model to explore morphological aspects of rippled beds. As far as sediment in suspension is concerned, Lagrangian particle tracking has generally been used in 2D oscillatory flow models (e.g. Hansen et al. (1994), Block et al. (1994), Perrier (1996), Andersen and Fredsøe (1999)).

Although 2D-models have achieved reasonable success in representing the main features of vortex dynamics and the associated sediment transport above rippled beds, these models are unduly complex from an engineering point of view. For the most part, existing engineering models attempt to represent ripples by simply enhancing the bed roughness \(k_s\) used in standard, one-dimensional vertical (1DV) ‘flat bed’ formulations. This approach has some merit for low ripples (e.g. Davies and Villaret, 2000), but has
severe conceptual limitations for steep ripples. Appropriate time-mean formulations for the eddy viscosity and sediment diffusivity above rippled beds, for use in a horizontally-averaged 1DV framework, have been proposed by Nielsen (1992) and Sleath (1991), and by Nielsen (1992), respectively. They presented empirical evidence to show that, in contrast to the plane bed case, a height-independent viscosity is appropriate in the near-bed vortex layer above ripples. Subsequently, it was shown by Davies and Villaret (1997, 1999) that the time-variation in eddy viscosity is more pronounced above ripples than above plane beds, with peaks in viscosity occurring near times of flow reversal.

In connection with sediment transport, the phase of sediment pick-up during the wave cycle is also significantly different above rippled beds, with pick-up being linked to the phase of vortex shedding. This has potentially important consequences for net sediment transport beneath asymmetrical waves and in combined wave and current flows; in both models and experiments (e.g. Murray et al., 1991) the important wave-related component of the net transport has been shown to be negative above rippled beds (i.e. offshore or opposed to the current, in the respective cases). This effect is not represented in existing engineering sand transport models. The motivation for the present study has therefore been to provide a relatively simple, 1DV modelling approach that includes an improved parameterisation of the near-bed vortex-layer above steep ripples, leading to improved practical predictions of local sand transport rates beneath waves, and also in combined wave-current flows.

RIPPLED BED MODEL

The model described here represents an extension of the 1DV turbulence-closure model of Davies and Li (1997). Its key new feature is an analytical, near-bed, sub-model that represents the processes of vortex shedding, and the associated entrainment of sediment at times of flow reversal. In this lower layer, the model solves the time-dependent, phase-averaged momentum equation for the horizontally-averaged (over a ripple wave-length) velocity, and the continuity equation for the suspended sediment concentration. In the upper layer, above the vortex-dominated region, the model reverts to the standard turbulence-closure formulation, subject to matching conditions for velocity, turbulent energy, eddy viscosity and sediment concentration, at a changeover level corresponding to two ripple-heights above the (undisturbed) mean bed level.

For waves alone, the model includes the following steps and assumptions:

- Momentum transfer in the near-bed layer is modelled using the ‘convective eddy viscosity’ of Davies and Villaret (1997, 1999). This time-varying, height-independent, eddy viscosity has maxima just ahead of times of flow reversal. Its mean value is determined using the formula proposed by Nielsen (1992) for very rough beds.

- In the continuity equation for suspended sediment, the diffusivity is taken as 4 times the eddy viscosity, as suggested by Nielsen (1992) on the basis of experimental data for rippled beds. This factor (the value of which has been optimised here) is needed in the vortex-dominated layer to account for 2D-3D spatial-temporal correlations between high concentration and locally upward velocity during the vortex shedding process.
The bottom boundary condition for sediment is a strongly time-varying pick-up function, which represents sediment entrainment associated with the vortex shedding process. This condition is imposed at the ripple crest level, the mean value of the pick-up function being based on Nielsen’s (1986) empirical reference concentration formula for ripples. The data from the Deltaflume referred to in the next section has been used to define the phase angle of sediment pick-up during the wave cycle. According to this data, peak suspended concentrations at the ripple crest level occur just ahead of flow reversal, and the phase of sediment pick-up has been defined to produce this outcome. The above procedures for waves alone have been extended for combined wave and current flows, as outlined later.

The modelling scheme described only becomes appropriate if the ripples are deemed to be ‘steep’, with height to wavelength ratio greater than about 0.12. If the ripple dimensions are not known from observation, the ripple height (\(\eta\)) and wavelength (\(\lambda\)) must be predicted at the outset using the hydrodynamic inputs and grain size composition of the bed. The approach adopted here was outlined by Davies and Villaret (2000); this involves use of the formulation of Wiberg and Harris (1994) for waves in isolation as the starting point to calculate \(\eta\) and \(\lambda\) in any given case. These ripples are predicted to be ‘orbital’, ‘sub-orbital’ or ‘anorbital’, depending upon the value of the ratio \(d_0/D_{50}\) (where \(d_0 = \text{orbital diameter}\)). To allow for the superimposition of a current on the waves, the orbital diameter \(d_0\) used in the above procedure has been replaced here by \(\alpha d_0\), where \(\alpha\) is given by the formula of Tanaka and Dang (1996). For simplicity, the waves have been treated for this purpose as sinusoidal with angle of attack (\(\phi\)) in the range \([0, \pi/2]\), and \(\alpha (\geq 1)\) has been based on the resolved component of the current in the wave direction. The bed roughness \((k_s)\) has then been determined from the standard rule \(k_s = 25\eta(\eta/\lambda)\).

The final consideration is the suspended sediment grain size \((D_s)\), which also may not be known from observation. Here, the size, and hence settling velocity, of the grains in suspension has been calculated as follows. The peak bed shear stress \(\tau_{wc}\) (skin friction) in the wave-current cycle and, hence, the (skin friction) friction velocity \(u'_{wc}\) have been estimated using Swart’s (1976) formula for the wave friction factor with \(k_s = 2.5D_{50}\). Then, based on a log-normal grain size distribution curve, the largest grain size in suspension (diameter \(D_{crit}\)) has been estimated by assuming that these grains have settling velocity \(W_{s,crit} = 0.8u'_{wc}\) (Fredsoe and Deigaard, 1992). From the resulting size \(D_{crit}\), the median diameter \(D_s\) of the grains in suspension, and its settling velocity \(W_{s,susp}\) have been estimated. The sediment in suspension may, if required, be further divided into size fractions (typically 3 or 5 fractions) based on the size distribution curve.

**COMPARISONS WITH EXPERIMENTAL DATA (WAVES ALONE)**

The two-layer model has been validated using data obtained in the Deltaflume of Delft Hydraulics. The large size of this flume (230 m long, 5 m wide and 7 m deep) allowed the wave and sediment transport phenomena to be studied at full scale. The measurements were made using an instrumented tripod platform ‘STABLE’ (Sediment
The instrumentation consisted of an acoustic backscatter system (ABS), with associated pumped sampling, and (ECM) velocity measurements at three heights above the bed. The ripples were measured using an acoustic ripple profiler (ARP). Full details of the experimental set up and instrumentation have been given by Thorne et al. (2002a,b).

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<th>Test</th>
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<th>T (s)</th>
<th>η (m)</th>
<th>λ (m)</th>
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* Wave Height (H) and Period (T), Ripple Height (η) and Wavelength (λ).

Fig. 1. Comparison between measured ripple dimensions (x) in the Deltaflume tests and the predictions of the (modified) formulation of Wiberg and Harris (1994).

The two-layer model has been compared with data from 10 tests carried out in the Deltaflume. The water depth was 4.5 m, and the bed comprised sand of median diameter $D_{50} = 0.329$ mm and geometric standard deviation $\sigma_g = [D_{84}/D_{16}]^{1/2} = 1.55$. The heights and periods of the regular, weakly-asymmetrical waves, and the
dimensions of the ripples measured using the ARP are shown in Table 1.

The analysis of the ABS results provided both horizontally (ripple-) averaged, time-mean, vertical profiles of concentration above the ripple crest level, and also the time-varying concentration structure above various locations on the ripple profile. The measured, phase and horizontally averaged, profiles of concentration have been compared with the model solutions. The clear temporal structure in the measurements has served to justify use of the present 1DV model formulation though, here, only time-mean aspects of the observations and model results are presented.

Figure 1 shows that the ripples in almost all of the Deltaflume tests were in the central ‘sub-orbital’ range. In this regime, ripple prediction schemes tend to perform less well than in the other regimes. Here, in order to provide the model with a reasonable overall representation of the ripple dimensions, Wiberg and Harris’ (1994) formulation has been modified in the orbital and sub-orbital regimes by the imposition of a maximum value of ripple steepness of 0.14. The resulting ripple dimensions have been used in the prediction of the cycle-mean, reference concentration at the ripple crest level. This is given by Nielsen’s (1986) formula, $C_0 = \gamma \theta_r^3$, where $\theta_r = \theta' / \{1 - \pi \eta / \lambda \}^2$ and $\theta'$ is the plane-bed (skin friction) Shields parameter. The results for $C_0$ are compared with the measured values (extrapolated to the ripple crest level from the ABS profiles, see Thorne et al., 2002a) in Figure 2. The empirical constant $\gamma$ in Nielsen’s formula has been re-scaled here from its standard value of 0.005 to the optimised of value 0.0022. This reduced value of $\gamma$ for the Deltaflume data reflects the proportion (10-40% by volume in the respective tests) of the bed material that was entrained into suspension.
Typical time-mean, horizontally-averaged, concentration profiles are shown for Tests 1, 4, 6 and 8 in Figure 3, in which the ABS results (crosses) are shown for vertical bins of height 10 mm. [The ABS was located at a height of 1.24 m above the bed.] The near-bed velocity amplitudes (fundamental component) in the four tests were 0.54, 0.65, 0.53 and 0.34 m/s, respectively; and the predicted settling velocities \( W_{s,susp} \) of the median grain size in suspension were 24.9, 29.2, 24.7 and 15.0 m/s. The model profiles show good overall agreement with the data in the lower vortex-dominated layer (thickness about 0.1 m), and also in the outer turbulent layer above this. But, in these and most other tests, systematic differences occur above a height of about 0.4 m.

The mean concentration profiles in Figure 3 were obtained with one grain size in suspension. However, the model can also include the effect of sediment size gradation. As noted earlier, the model determines initially the maximum grain size that can be lifted into suspension. For the case of Test 3, only about 15% (by volume) of the grains comprising the bed were capable of being suspended. This is illustrated in Figure 4 in which the observed bed grain size distribution (crosses) is represented by the log-normal curve, and the full vertical line indicates the critical (maximum) size \( D_{crit} \) of grains in suspension \( D_{crit} = 0.208 \text{ mm} \), based on the peak value of Shields parameter (skin friction) during the cycle. The dashed lines show the subdivision into five volumetrically equal fractions of the grains with size less than \( D_{crit} \). Each of these fractions has been treated separately by the model in the following figures.
A consequence of using a multiple-grain approach is that height-variation in the median size of the suspended material is obtained as part of the solution. For Test 3, the results in Figure 5 show that the model predicts quite well both the overall grain size...
of the sediment in suspension (which is much smaller than the $D_{50}$ of the bed material), and also the decrease in grain size with height above the bed. The five size fractions modelled correspond to the vertical dashed lines, and the measured grain size in suspension (from pumped samples) is shown by the crosses.

![Graph showing concentration profile](image)

**Fig. 6.** Mean concentration profile made up of the sum of 5 grain size fractions for the case of Test 3. The corresponding profile based on one fraction is also shown.

The mean C-profiles for the five size fractions in suspension are shown for Test 3 in Figure 6. Each fraction is assumed to have the same reference concentration at the ripple crest level. Above this the respective fractions exhibit rather different behaviour, with relatively large concentrations occurring for the smaller fractions, and vice versa. The slope of the total mean C-profile obtained by summing the five fractions agrees quite well with the measurements through the bottom 50 cm of the flow. The effect of using a graded size approach, as opposed to assuming just one grain size in suspension (c.f. Figure 3), can seen here by the difference between the C-profiles labelled ‘1 fraction’ and ‘5 fractions’. The difference between the two C-profiles is not particularly great for this (or any of the other) Delta Flume tests, amounting to a factor of only about 2 in the suspended concentration at 50 cm above the bed. This is due to the fact that only a small proportion of the coarse bed material is found in suspension for Test 3, and so there is only a small contrast between the settling velocities of the five fractions (the grain sizes of which are shown in the legend). In situations in which a greater proportion of the bed material is suspended, use of multiple grain fractions has a far more significant effect on the final result.
APPLICATION OF THE MODEL IN COMBINED WAVE AND CURRENT FLOWS

The formulation of the eddy viscosity for waves alone has been extended to allow the model to be run in situations involving combined waves and currents. As for waves alone, the model initially predicts the ripple dimensions, to ensure that the steepness ($\eta/\lambda$) is sufficiently large for use of the rippled bed model. Next, a combined wave-current, analytical, eddy viscosity is defined for use in the near-bed vortex-dominated layer (again assumed to be of thickness 2 ripple heights). Here, the approach of Sleath (1991) has been adopted. This involves a simple, linear, superimposition of the earlier height-independent eddy viscosity ($K_w$) for the waves alone, and a height-varying viscosity ($K_c$) for the current alone. The velocity scale used to characterise the current contribution ($K_c \sim u_{*c}z$) has been taken as the friction velocity $u_{*c}$ for the current alone. This has been determined taking into account the angle of wave attack on the current. In collinear wave-current cases, $u_{*c}$ has been estimated on the basis of the ripple roughness ($k_s = 25\eta(\eta/\lambda)$) while, in perpendicular cases, it has been estimated using the grain roughness ($k_s = 2.5D_{50}$). Since the flow in the latter case is aligned with the ripple troughs and crests, rather than being from trough to crest, the friction velocity is expected to be correspondingly smaller. In general angular cases, the two limiting values of $u_{*c}$ have been implemented in a weighted-average expression to provide the required velocity scale.

The resulting combined ‘constant + linear’ eddy viscosity structure in the vortex dominated layer has been implemented in the numerical model as discussed earlier. Matching conditions on velocity, turbulent energy, eddy viscosity, and concentration, have again been applied at height $2\eta$, and above this the standard ‘flat bed’ turbulence scheme has been run for the required wave-current combination. Interestingly, the ‘constant + linear’ eddy viscosity structure in the vortex dominated layer produces a mean $C$-profile that is mathematically equivalent to the ‘convective’ $C$-profile proposed, on the basis of rather different physical arguments, by Nielsen (1992) for waves alone (see Thorne et al., 2002a).

As far as practical predictions in wave-current cases are concerned, the overall effect of the rippled bed sub-model is to enhance sediment diffusion into suspension, thereby increasing suspended concentrations and transport rates. This is illustrated here with reference to a set of data obtained at Maplin Sands, Outer Thames Estuary, UK, where low, short period waves were combined with relatively strong currents, over a range of angles of wave attack, above a bed of very fine sand with $D_{50} = 0.115$ mm (Whitehouse et al., 1996). In 7 of the 18 cases considered, the bed was predicted to comprise steep ripples, and so the rippled bed model was implemented. In Figure 7a, the results obtained with the rippled bed model are compared with sediment concentrations (pumped samples) measured at two heights above the bed (0.05 m and 0.1 m). In Figure 7b, the same comparison is shown for results obtained with the ‘flat bed’ model run with an enhanced bed roughness ($k_s$) based on the predicted ripple dimensions. The results obtained with the rippled bed model are generally more convincing, the predicted concentration at both heights being increased substantially in most tests by use...
of the rippled bed model. As might be expected, on account of the increase in concentration, the predicted transport rate also increases. However, no transport measurements were made for comparison in the present Maplin cases.

![Graphs](image_url)

Figure 7. Maplin Sands tests: (a) comparison between measured and predicted (Rippled bed model) suspended sediment concentrations at heights 0.05m (o) and 0.1m (x) above the bed; (b) equivalent comparison between the measured concentrations and TKE Flat bed model. Perfect agreement is indicated by the full 45° line, and factor ±10 agreement by the dashed lines.

CONCLUSIONS

On the basis of an analysis of horizontally-averaged acoustic backscatter (ABS) concentration data from the Deltaflume, it seems justifiable to represent sediment in suspension above ripples using a one-dimensional vertical (1DV) model. The complex motions involved in vortex formation and shedding in the near-bed layer above the ripples have been represented here by means of a strongly time-varying eddy viscosity, with peaks at about the time of flow reversal in the wave cycle. The associated time-varying sediment pick-up rate from the bed has been linked to the phase angle of eddy shedding. In addition, the sediment diffusivity has been enhanced in comparison with the eddy viscosity. Above the near-bed vortex dominated layer, the model reverts to a standard, One-equation, turbulence-closure formulation.

The new, two-layer, ‘rippled bed’ model makes reasonable predictions of i) the observed ripple dimensions, ii) the reference concentration and mean concentration profiles, iii) the suspended sediment grain size in relation to the grain size of the bed material, and its variation with height above the bed. In addition, intra-wave processes, which differ qualitatively from those above a plane bed, are modelled satisfactorily (though this has not been presented here for brevity). The present model treats, if required, multiple grain fractions in suspension.

The ‘rippled bed’ model has been validated for waves alone using data obtained in the Deltaflume of Delft Hydraulics. It has been used also in comparisons with field data
obtained in wave-current flows. Use of the model in field situations involving steep ripples provides: i) improved predictions of suspended sediment concentrations; and ii) a general enhancement in sediment transport rates. However, whether a significant improvement in transport predictions can yet be claimed is unclear, due to the limited amount of experimental (field) evidence available in the steeply rippled bed regime. Further validation of the model, particularly in combined wave and current cases, is required. However, it is clear that the processes represented by the new model have potential importance for the accurate prediction of suspended concentrations and net transport rates.

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REFERENCES


